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ON DOMINATION NUMBER AND ECCENTRIC CONNECTIVITY INDEX OF TRANSFORMATION GRAPHS

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Abstract: A set $D \subseteq V$ is a dominating set of G if every vertex in $V - D$ is adjacent to one or more vertices in D. The minimum cardinality of a dominating set of a graph G is called the domination number of G and is denoted by $\gamma(G)$. The eccentric connectivity index is the sum of the product of eccentricity and degree of every vertex in G. In this paper, we present upper bounds for the total transformation graphs in terms of domination number and eccentric connectivity index of a graph G.

Keywords: Domination Number; eccentricity connectivity index; transformation graph.

AMS Subject Classification: 05C69;05C92.

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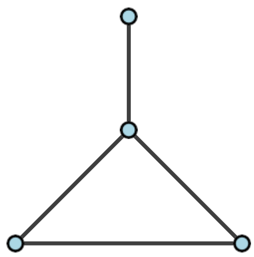
1 Introduction

Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$. The degree of a vertex $v \in G$ is denoted by $deg_G(v)$ and defined as $deg_G(v) = |\{u \setminus uv \in E(G)\}|$. The eccentricity of a vertex $v \in G$ is the largest distance between v and u for some $u \in V(G)$. A set $D \subseteq V$ is a dominating set of G if every vertex in $V - D$ is adjacent to one or more vertices in D. The minimum cardinality of a dominating set of a graph G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set D with minimum cardinality is called $\gamma(G)$-set of G. Further, a minimal dominating set is a dominating set in a graph that is not a proper subset of any other dominating set. We follow [5] for unexplained terminology and notation. Sharma et.al., [13] have put forward a novel topological index namely, the eccentric-connectivity index $ECI(G)$ of a molecular graph G. Which is defined as follows:

$$ECI(G) = \sum_{i=1}^{n}(e(v_i)deg(v_i)). \qquad (1)$$

For more details on topological indices refer [3, 8, 9, 14, 15]. Wu and Meng [16] have generalized the total graph by defining the following transformation graphs:



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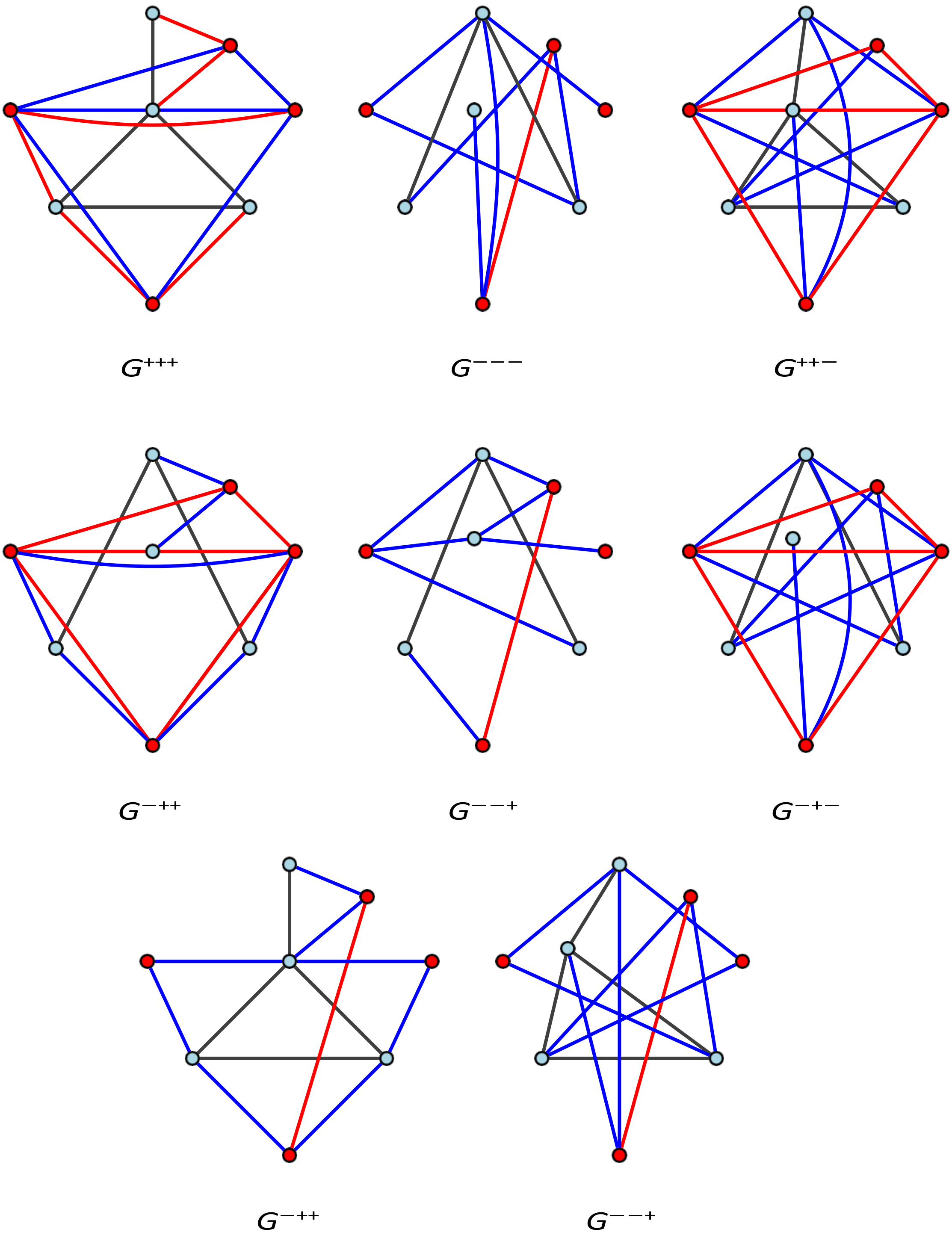

Figure 1. A graph G and its transformation graphs.

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2 Results

In this section we obtain upper bounds for domination number together with eccentric-connectivity index of total transformation graphs in terms of order, size and the first Zagreb index.

Theorem 2.1. Let $G = (n, m)$ be any connected graph. Then

$$\gamma(G^{+++}) + ECI(G^{+++}) \leq 2ECI(G) + M_1(G) + \frac{9m+n}{2} + \xi(G) \quad (2)$$

where $\xi(G) = \sum_{u_ju_k \in E(G)} e_G(u_j)(deg_G(u_j) + deg_G(u_k))$.

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{+++}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{+++})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$. Therefore,

$$\gamma(G^{+++}) \leq \frac{m+n}{2}. \quad (3)$$

Further,

$$\begin{aligned}
ECI(G^{+++}) &= \sum_{u_i \in V(G^{+++}) \cap V(G)} e_{G^{+++}}(u_i) \cdot deg_{G^{+++}}(u_i) + \\
&\quad \sum_{u_j \in V(G^{+++}) \cap E(G)} e_{G^{+++}}(u_j) \cdot deg_{G^{+++}}(u_j) \\
&\leq \sum_{u_i \in V(G)} \left[(e_G(u_i)+1) \cdot 2deg_G(u_i)\right] + \\
&\quad \sum_{u_j, u_k \in E(G)} \left[(e_G(u_j)+1) \cdot (deg_G(u_j) + deg_G(u_k))\right] \\
&= 2\sum_{u_i \in V(G)} e_G(u_i) deg_G(u_i) + 2\sum_{u_i \in V(G)} deg_G(u_i) \\
&\quad + \sum_{u_ju_k \in E(G)} e_G(u_j)(deg_G(u_j) + deg_G(u_k)) + \sum_{u_ju_k \in E(G)} (deg_G(u_j) + deg_G(u_k)) \\
&\leq 2ECI(G) + M_1(G) + 4m + \xi(G), \quad (4)
\end{aligned}$$

Hence by (3) and (4) the result follows. □

Theorem 2.2. Let $G = (n, m)$ graph. Then

$$\gamma(G^{---}) + ECI(G^{---}) \leq 3(m+n)(m+n-1) + \frac{(m+n)}{2} - 12m - 3M_1(G).$$

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Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{---}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{---})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$. Therefore,

$$\gamma(G^{---}) \leq \frac{m+n}{2}. \tag{5}$$

Further,

$$\begin{aligned} ECI(G^{---}) &= \sum_{u_i \in V(G^{---}) \cap V(G)} e_{G^{---}}(u_i) \cdot deg_{G^{---}}(u_i) + \\ &\quad \sum_{u_j \in V(G^{---}) \cap E(G)} e_{G^{---}}(u_j) \cdot deg_{G^{---}}(u_j) \\ &\leq \sum_{u_i \in V(G)} \left[3 \cdot (m + n - 1 - 2(deg_G(u_i)))\right] \\ &+ \sum_{u_j, u_k \in E(G)} \left[3 \cdot (m + n - 1 - (deg_G(u_j) + deg_G(u_k)))\right] \\ &= 3(m + n - 1)n - 6 \sum_{u_i \in V(G)} (deg_G(u_i)) \\ &+ 3(m + n - 1)m - 3 \sum_{u_j, u_k \in E(G)} (deg_G(u_j) + deg_G(u_k)) \\ &\leq 3(m + n)(m + n - 1) - 12m - 3M_1(G). \end{aligned} \tag{6}$$

Hence by (5) and (6) the result follows. □

Theorem 2.3. Let $G = (n, m)$ graph. Then

$$\gamma(G^{++-}) + ECI(G^{++-}) \leq 4mn + 4m(n - 4) + \frac{(m+n)}{2} + 4M_1(G).$$

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{++-}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{++-})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$ Therefore,

$$\gamma(G^{++-}) \leq \frac{m+n}{2}. \tag{7}$$

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Further,

$$\begin{aligned} ECI(G^{++-}) &= \sum_{u_i \in V(G^{++-}) \cap V(G)} e_{G^{++-}}(u_i) \cdot deg_{G^{++-}}(u_i) + \\ & \quad \sum_{u_j \in V(G^{++-}) \cap E(G)} e_{G^{++-}}(u_j) \cdot deg_{G^{++-}}(u_j) \\ &\leq \sum_{u_i \in V(G)} [4m] + \sum_{u_j, u_k \in E(G)} [4 \cdot (n - 4 + (deg_G(u_j) + deg_G(u_k)))] \\ &\leq 4mn + 4m(n-4) + 4M_1(G). \qquad (8) \end{aligned}$$

Hence by (7) and (8) the result follows. □

Theorem 2.4. Let $G = (n, m)$ graph. Then

$$\gamma(G^{--+}) + ECI(G^{--+}) \leq 3n(n+1) - 6m + 3m(m+3) + \frac{(m+n)}{2} - 3M_1(G).$$

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{--+}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{--+})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$. Therefore,

$$\gamma(G^{--+}) \leq \frac{m+n}{2}. \qquad (9)$$

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Further,

$$\begin{aligned} ECI(G^{--+}) &= \sum_{i=1}^{n} e_{G^{--+}}(u) \cdot deg_{G^{--+}}(u) \\ &= \sum_{u_i \in V(G^{--+}) \cap V(G)} e_{G^{--+}}(u_i) \cdot deg_{G^{--+}}(u_i) + \\ & \quad \sum_{u_j \in V(G^{--+}) \cap E(G)} e_{G^{--+}}(u_j) \cdot deg_{G^{--+}}(u_j) \\ &\leq \sum_{u_i \in V(G)} [3 \cdot (n + 1 - (deg_G(u_i)))] \\ & \quad + \sum_{u_j, u_k \in E(G)} [3 \cdot (m + 3 - (deg_G(u_j) + deg_G(u_k)))] \\ &\leq 3n(n+1) - 6m + 3m(m+3) - 3M_1(G). \qquad (10) \end{aligned}$$

Hence by (9) and (10) the result follows. □

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Theorem 2.5. Let $G = (n, m)$ graph. Then

$$\gamma(G^{+-+}) + ECI(G^{+-+}) \leq 12m + 3m(m+3) + \frac{(m+n)}{2} - 3M_1(G).$$

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{+-+}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{+-+})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$ Therefore,

$$\gamma(G^{+-+}) \leq \frac{m+n}{2}. \tag{11}$$

Further,

$$\begin{aligned} ECI(G^{+-+}) &= \sum_{i=1}^{n} e_{G^{+-+}}(u) \cdot deg_{G^{+-+}}(u) \\ &= \sum_{u_i \in V(G^{+-+}) \cap V(G)} e_{G^{+-+}}(u_i) \cdot deg_{G^{+-+}}(u_i) + \\ &\quad \sum_{u_j \in V(G^{+-+}) \cap E(G)} e_{G^{+-+}}(u_j) \cdot deg_{G^{+-+}}(u_j) \\ &\leq \sum_{u_i \in V(G)} \left[3 \cdot (2(deg_G(u_i)))\right] \\ &+ \sum_{u_j, u_k \in E(G)} \left[3 \cdot (m + 3 - (deg_G(u_j) + deg_G(u_k)))\right] \\ &\leq 12m + 3m(m+3) - 3M_1(G). \end{aligned} \tag{12}$$

Hence by (11) and (12) the result follows. □

Theorem 2.6. Let $G = (n, m)$ graph. Then

$$\gamma(G^{-+-}) + ECI(G^{-+-}) \leq 3n(m+n-1) + 3m(n-4) - 12m + \frac{(m+n)}{2} + 3M_1(G).$$

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{-+-}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{-+-})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$. Therefore,

$$\gamma(G^{-+-}) \leq \frac{m+n}{2}. \tag{13}$$

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Further,

$$\begin{aligned} ECI(G^{-+-}) &= \sum_{i=1}^{n} e_{G^{-+-}}(u)\cdot deg_{G^{-+-}}(u) \\ &= \sum_{u_i\in V(G^{-+-})\cap V(G)} e_{G^{-+-}}(u_i)\cdot deg_{G^{-+-}}(u_i) + \\ &\quad \sum_{u_j\in V(G^{-+-})\cap E(G)} e_{G^{-+-}}(u_j)\cdot deg_{G^{-+-}}(u_j) \\ &\leq \sum_{u_i\in V(G)} [3\cdot(m+n-1-2(deg_G(u_i)))] \\ &+ \sum_{u_j,u_k\in E(G)} [3\cdot(n-4+(deg_G(u_j)+deg_G(u_k)))] \end{aligned}$$

$$\leq 3n(m+n-1)+3m(n-4)-12m+3M_1(G). \tag{14}$$

Hence by (13) and (14) the result follows. □

Theorem 2.7. Let $G = (n, m)$ graph. Then

$$\gamma(G^{-++}) + ECI(G^{-++}) \leq 3n(n-1) + \frac{(m+n)}{2} + 3M_1(G).$$

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{-++}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{-++})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$ Therefore,

$$\gamma(G^{-++}) \leq \frac{m+n}{2}. \tag{15}$$

Further,

$$\begin{aligned} ECI(G^{-++}) &= \sum_{i=1}^{n} e_{G^{-++}}(u)\cdot deg_{G^{-++}}(u) \\ &= \sum_{u_i\in V(G^{-++})\cap V(G)} e_{G^{-++}}(u_i)\cdot deg_{G^{-++}}(u_i) + \\ &\quad \sum_{u_j\in V(G^{-++})\cap E(G)} e_{G^{-++}}(u_j)\cdot deg_{G^{-++}}(u_j) \\ &\leq \sum_{u_i\in V(G)} [3\cdot(n-1)] \\ &+ \sum_{u_j,u_k\in E(G)} [3\cdot(deg_G(u_j)+deg_G(u_k))] \end{aligned}$$

https://doi.org/10.71058/jodac.v10i06008

$$\leq \quad 3n(n-1) + 3M_1(G). \tag{16}$$

Hence by (15) and (16) the result follows. □

Theorem 2.8. Let $G = (n, m)$ graph. Then

$$\gamma(G^{+--}) + ECI(G^{+--}) \leq 4mn + 4m(m+n-1) - \frac{(m+n)}{2} + 4M_1(G).$$

Proof. Let $G = (V, E)$ be a graph with $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$. Then $V(G^{+--}) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_m\}$. Clearly $|V(G^{+--})| = m + n$. Since for any connected graph G, $\gamma(G) \leq \frac{n}{2}$. Therefore,

$$\gamma(G^{+--}) \leq \frac{m+n}{2}. \tag{17}$$

Further,

$$\begin{aligned}
ECI(G^{+--}) &= \sum_{i=1}^{n} e_{G^{+--}}(u) \cdot deg_{G^{+--}}(u) \\
&= \sum_{u_i \in V(G^{+--}) \cap V(G)} e_{G^{+--}}(u_i) \cdot deg_{G^{+--}}(u_i) + \\
&\quad \sum_{u_j \in V(G^{+--}) \cap E(G)} e_{G^{+--}}(u_j) \cdot deg_{G^{+--}}(u_j) \\
&\leq \sum_{u_i \in V(G)} [4 \cdot (m)] \\
&\quad + \sum_{u_j, u_k \in E(G)} [4 \cdot (m + n - 1 - (deg_G(u_j) + deg_G(u_k)))]
\end{aligned}$$

$$\leq \quad 4mn + 4m(m+n-1) - 4M_1(G). \tag{18}$$

Hence by (17) and (18) the result follows. □

3 Conclusion

In this paper, we have obtained bounds for all eight transformation graphs in terms of domination number and eccentric connectivity index of a graph G.

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