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Abstract: *Sandwich composite structures are widely employed in automotive, aerospace and marine industries due to their exceptional stiffness-to-weight ratio, strength-to-weight ratio, vibration attenuation, enhanced aero-elastic characteristics and energy absorption capabilities. Accurate prediction of their dynamic behavior under damaged conditions remains a major challenge because conventional classical models fail to capture high-frequency dispersion effects, while pure wave-based models often neglect realistic boundary conditions. This study presents a unified hybrid modeling framework based on classical vibration theory with wave propagation analysis to evaluate the dynamic response of Glass Fiber-Reinforced Polymer (GFRP) sandwich panels across a wide frequency range. The hybrid frequency-domain equations were numerically implemented using Python to extract frequency-amplitude response enabling efficient computation of broadband Frequency Response Functions (FRFs). Experimental modal testing was conducted under Clamped-Free-Free-Free (cantilever) boundary conditions for undamaged, core-damaged, delaminated, and impact-damaged specimens. The numerically predicted FRFs were systematically correlated with experimental results. The results indicate that the hybrid models accurately capture resonant peak shifts, damping variations, and stiffness degradation effects with significantly reduced prediction error. The proposed framework provides a robust and computationally efficient tool for dynamic analysis and reliable vibration-based structural health monitoring of sandwich composite structures.*

Keywords: *Sandwich Structures, Hybrid Modeling, Wave Propagation, Structural Health Monitoring.*

1.0 Introduction

A typical sandwich configuration comprising of a lightweight compliant core and two stiff face sheets bonded together which delivers superior bending rigidity while minimizing structural mass. The above properties make them ideal for vibration-sensitive and load-bearing environments [1, 2].

Despite these advantages, accurately predicting the dynamic characteristics of sandwich structures is still a major challenge. Their multi-layered and heterogeneous nature introduces material discontinuities and interfacial phenomena that complicate vibration and wave propagation characteristics [3]. Moreover, core materials typically possess low shear modulus and high damping capacity, leading to pronounced energy dissipation and shear deformation-especially at higher frequencies [4].

Over the years, various numerical and analytical models have been proposed to characterize these dynamics. Classical Laminate Theory (CLT) and First-Order Shear Deformation Theory (FSDT) are generally adopted. While CLT is computationally efficient for low-frequency analysis, it neglects transverse shear deformation, making it unsuitable for thick sandwich constructions [5]. FSDT, along with refined models like Timoshenko beam theory, incorporates shear deformation and rotary inertia effects, providing improved predictions in such cases [6,7]. However, even these enhanced classical models often oversimplify through-thickness stress distributions and struggle to accurately capture localized or high-frequency responses.

To address these shortcomings, wave-based modeling techniques have emerged as powerful tools, particularly in high-frequency regimes. The Spectral Finite Element Method (SFEM) utilizes exact solutions of governing differential equations as shape functions in the frequency domain, enabling highly accurate dynamic predictions with relatively few degrees of freedom. It has been effectively implemented in periodic sandwich and composite beams [8,9]. Similarly, the Wave Finite Element Method (WFEM) incorporates structural periodicity through Floquet-Bloch theory, making it effective for wave propagation analysis in ribbed and composite structures [10].

The Semi-Analytical Finite Element (SAFE) method is also noteworthy, combining finite element discretization in the cross-section with analytical wave solutions along the axial direction. It is particularly effective in waveguide-type problems such as rails, plates, and sandwich structures, enabling efficient computation of dispersion relations and wave modes [11]. However, wave-based models typically assume infinite or semi-infinite domains and often neglect the influence of boundary conditions, rendering them insufficient for finite-sized, real-world structures with complex constraints.

Recognizing the complementary strengths of classical and wave-based methods, hybrid modeling strategies have emerged as promising alternatives. These approaches aim to integrate the low-frequency accuracy of classical theories with the high-frequency resolution of wave-based models, thereby enabling comprehensive dynamic analysis over a broad frequency spectrum [12]. For example, Mahapatra and Gopalakrishnan [13] proposed a hybrid wavelet-spectral finite element approach to capture both time and space-localized responses in composite beams. Other researchers have employed multiscale hybrid frameworks; wherein different parts of the structure are analyzed using distinct physical models depending on their spatial or frequency sensitivity [14].

Although both analytical and experimental approaches have been independently investigated, minimal research has been conducted on experimentally validated hybrid frameworks that integrate global modal behavior with local wave effects. Therefore, the objective of this investigation is to develop and experimentally validate a classical, pure-wave based modeling and hybrid approach capable of predicting the dynamic behavior of damaged GFRP sandwich structures over a wide frequency range.

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2.0 Theoretical Formulation

2.1 Classical Modeling Approach

Classical modeling techniques provide a foundational understanding of the structural behavior of sandwich constructions under dynamic loads. In the framework of vibration analysis, these methods are well-suited for predicting global natural frequencies and mode shapes in the low-frequency regime. Among these, Classical Sandwich Theory (CST) and First-Order Shear Deformation Theory (FSDT) are widely used for analyzing symmetric sandwich composite structures with isotropic or orthotropic layers.

The CST**,** first proposed by Allen [15], assumes that the face sheets of the structure carry all in-plane and bending stresses. The core of the structure is soft in the axial direction but stiff in shear, resisting transverse shear loads. Perfect bonding exists between the core and the face sheets, with no interfacial slip. Under these assumptions, a sandwich beam or plate behaves like an equivalent homogeneous structure with modified stiffness properties. The governing equation for a simply supported sandwich beam under uniform bending loading is:

$$\mathrm{M} = EI\frac{d^2w(x)}{dx^2} \tag{1}$$

The momentum equilibrium yields $\frac{d^2M(x)}{dx^2} = EI\frac{d^4w(x)}{dx^4}$ (2)

The term represents bending resistance.

In sandwich structures, the transverse shear force is:

$$\mathrm{V(x)} = k_s\frac{dw(x)}{dx} \tag{3}$$

And the equilibrium of forces in the vertical direction (Euler-Bernoulli extended with shear) gives:

$$\frac{dV(x)}{dx} = k_s\frac{d^2w(x)}{dx^2} \tag{4}$$

This term represents shear resistance from the core**.**

Total Equilibrium Equation

Apply Newton's second law (static equilibrium with distributed load q(x)):

$$\frac{d^2M(x)}{dx^2} + \frac{dV(x)}{dx} = q(x) \tag{5}$$

$$EI\ \frac{d^4w(x)}{dx^4} + \ k_s\frac{d^2w(x)}{dx^2} = q(x) \tag{6}$$

Where w(x) is the transverse deflection, k_s is the shear stiffness of the core.

The bending stiffness D is computed as:

$$D = 2\ E_f bt_f\left(\frac{h}{2}\right)^2 + \frac{E_C bh^3}{12} \tag{7}$$

with E_f and E_c as Young's moduli of face sheet and core, t_f face sheet thickness, h the total thickness b is the width of the sandwich beam.

The shear stiffness $k_{s,}$ of the core is defined as:

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$$k_s = \frac{bG_c h_c}{\phi} \quad (8)$$

Where G_c is the shear modulus of the core, h_c is the core thickness, and ϕ is the shear correction factor (typically 1.2 for rectangular cores).
Natural frequencies are estimated using free vibration analysis, the governing equation becomes [6]:

$$D\,\frac{d^4 w(x,t)}{dx^4} + k_s \frac{d^2 w(x,t)}{dx^2} + \rho A\,\frac{d^2 w(x,t)}{dt^2} = 0 \quad (9)$$

Applying separation of variables, w(x,t)=W(x) cos (ωt) and the boundary condition is assumed as simply supported, the solution yields:

$$\omega_n = \sqrt{\frac{D\left(\frac{n\pi}{L}\right)^4 + k_s\left(\frac{n\pi}{L}\right)^2}{\rho A}} \quad (10)$$

Where ω_n is the natural frequency of mode n, ρ is the density and A is the cross-sectional area. The deflection of the sandwich beam is given by:

$$\delta = \frac{5qL^4}{384\,D} + \frac{qL^2}{8S} \quad (11)$$

Combines bending and shear deflection components for accurate analysis.

2.2 Pure Wave-Based Model

Wave propagation in sandwich structures is governed by the interplay between their layered configuration, material heterogeneity, and geometric discontinuities [17, 18]. In such media, waves experience dispersion, attenuation, and mode conversion, making the prediction of dynamic response complex. This section presents the mathematical modeling of wave propagation based on fundamental principles of continuity and equilibrium, followed by the derivation of dispersion relations and an analysis of wave interactions at layer interfaces.

2.2.1 Derivation of Wave Equations in Layered Media

To derive the wave equations in a layered sandwich structure, an infinitely long, isotropic elastic layer of thickness h, density ρ, Young's modulus E and shear modulus G. Let the structure support longitudinal and transverse (flexural and shear) waves.

a) Longitudinal Wave Motion

Assuming 1D axial displacement u(x,t) in the x-direction, the axial force N = EA$\frac{\partial u}{\partial x}$
From Newton's second law

$$\rho A\,\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(EA\,\frac{\partial u}{\partial x}\right) \quad (12)$$

E and A are constant over the segment

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2} \quad (13)$$

This is the 1D wave equation, whose solution is:

$$\boldsymbol{u(x,t) = U\,e^{i\,(kx-\omega t)}} \quad \mathbf{(14)}$$

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Where ω is angular frequency (rad/s), k is wave number (rad/m), $c = \sqrt{\frac{E}{\rho}}$ is the phase velocity

(b) Transverse Flexural Wave Motion (Euler-Bernoulli Beam)

For transverse displacement w(x,t), considering bending stiffness D = EI

$$\rho\, A\, \frac{\partial^2 w}{\partial t^2} + D\, \frac{\partial^4 \omega}{\partial x^4} = 0 \tag{15}$$

Substituting, a harmonic wave solution: $\boldsymbol{w(x,t) = W\, e^{i\,(kx-\omega t)}}$

Leads to the dispersion relation: $\omega^2 = \frac{D}{\rho A} k^4 \Rightarrow \omega = \sqrt{\frac{D}{\rho A}}\; k^2$ (16)

This shows dispersive behavior and phase velocity depends on frequency:

$$c_p = \frac{\omega}{k} \sqrt{\frac{D}{\rho A}}\;\; k \tag{17}$$

(c) Shear Deformation (Timoshenko Beam Model)

For higher frequency behavior or thick sandwich beams, shear deformation and rotary inertia are included:

$$\rho\, A\, \frac{\partial^2 w}{\partial t^2} = \frac{\partial Q}{\partial x},\;\; Q = kGA\left(\frac{\partial w}{\partial x} - \emptyset\right) \tag{18}$$

$$I\rho\, \frac{\partial^t \emptyset}{\partial t^2} = \frac{\partial M}{\partial x} \qquad M = EI\, \frac{\partial \emptyset}{\partial x} \tag{19}$$

where: ϕ(x,t) is the rotation of the cross-section, Q is shearing force, M is bending moment, k is shear correction factor. These equations lead to coupled wave equations and more accurate dispersion relations that better model sandwich structures.

2.2.2 Dispersion Relations and the Phase Velocity

In layered media like sandwich structures, dispersion arises because of the interplay between the stiff face sheets and compliant core. Each layer supports different wave speeds due to differing stiffness and density.

Assuming harmonic wave solutions for displacements:

$\boldsymbol{u(x,t) = U\, e^{i\,(kx-\omega t)}}$ and $\boldsymbol{w(x,t) = W\, e^{i\,(kx-\omega t)}}$ (20)

The dispersion relation for each mode is derived by substituting into the governing equations and applying boundary and continuity conditions.

For a multilayer system, dispersion relations are typically obtained by solving matrix eigenvalue problems, such as:

$$[K(\omega) - k^2 M(\omega)]\vec{u} = 0 \tag{21}$$

The determinant condition**:**

Det $[K(\omega) - k^2 M(\omega)] = 0$ (22)

yields allowed values of k for each frequency ω, characterizing propagating modes and their speeds.

Phase velocity $c_p = \frac{\omega}{k}$ (23)

Group velocity: $c_g = \frac{\partial w}{\partial k}$ (24)

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In practice, dispersion curves ω(k) are plotted numerically to understand modal propagation in sandwich beams and plates.

2.2.3 Wave Reflection, Transmission, and Mode Conversion at Interfaces

When waves encounter a discontinuity, such as a change in layer properties, they undergo:

- Reflection (R): Part of the wave is reflected back.
- Transmission (T): Part of the wave continues into the next layer.
- Mode conversion: A longitudinal wave may generate transverse or shear waves upon hitting an interface.

Interface Conditions:
At an interface between two layers (say layer 1 and layer 2), the following must be satisfied:

- Continuity of displacement:
  $u_1 = u_2$ and $w_1 = w_2$
  Continuity of stress (force/moment):
  $\sigma_1 = \sigma_2$ and $M_1 = M_2$

Using these conditions, reflection and transmission coefficients can be computed by matching wave fields across interfaces.

Let an incident wave in layer 1 be:

$$u^{(1)} = A_i e^{i(k_1 x - \omega t)} + A_r e^{i(k_1 x - \omega t)} \quad (25)$$

By applying boundary conditions at the interface (say at x = 0), a linear system in A_r and A_t is formed and solved for reflection $R = \frac{A_r}{A_i}$ and transmission $T = \frac{A_t}{A_i}$.

These coefficients depend on impedance mismatch: $Z_i = \rho_i c_i$, (wave impedance). A large mismatch between layers leads to stronger reflections and potential mode conversions, which must be accurately captured in high-fidelity wave models for sandwich structures.

2.3 Hybrid Integration

While classical models excel at predicting global responses at lower frequencies and pure wave-based models offer precise insights at higher frequencies, neither alone can fully characterize the dynamic behavior of real-world sandwich structures over a broad frequency range. To bridge this gap, a hybrid modeling strategy is proposed. This section presents a formulation that integrates modal displacements from classical analysis with local wave amplitudes from wave propagation models using impedance-based coupling and frequency-domain representation.

2.3.1 Coupling of Modal Displacements with Local Wave Amplitudes

In Hybrid Model (HM), the structure is partitioned into two conceptual regions:

- Global domain: modeled using CST, FSDT or similar plate/beam formulations.
- Local domain (interface/substructure): modeled using wave propagation methods [18,19].

The modal displacement field in the global domain is expressed as:

$$\omega_g(x,t) = \sum_{n=1}^{N} \Phi_n(x) q_n(t) \quad (26)$$

Where $\Phi_n(x)$ is the n^{th} mode shape from classical modal analysis and $q_n(t)$ is the time-dependent modal amplitude. The wave field in the local domain is expressed in terms of forward and backward propagating waves:

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$\omega_l(x,t) = \sum_{m=1}^{M} A_m^+ e^{i(k_m x - \omega t)} + A_m^- e^{-i(k_m x - \omega t)}$ (27)

where: A_m^+ and A_m^- are complex wave amplitudes for mode m,

- k_m is the wave number for mode m,
- ω is the angular frequency.

Coupling Condition: Displacement and Force Continuity
At the interface between global and local domains (e.g., at $x = x_0$):

- **Displacement continuity:**
  $w_g(x_0,\omega) = w_l(x_0,\omega)$

- **Shear force continuity:**
  $Q_g(x_0,\omega) = Q_l(x_0,\omega)$

The modal shear force in the classical domain (e.g., from CST or FSDT):

$Q_g(x_0,\omega) = \sum_n \big(K_n \Phi'_n(x_0)\, q_n(\omega)\big)$ (28)

The wave-based shear force can be derived from the wave impedance and amplitude:

$Q_l(x_0,\omega) = \sum_m Z_m \left(A_m^+ e^{ik_m x_0} - A_m^- e^{-ik_m x_0}\right)$ (29)

These conditions yield a system of equations to solve for unknown modal amplitudes $q_n(\omega)$ and wave amplitudes $A_m^{\pm}$, enabling a seamless transition between models.

2.3.2 Impedance Matching and Energy Transfer Methods

To ensure energy conservation and smooth transition across domains, mechanical impedance plays a key role.

Mechanical Impedance Definition:
For a wave propagating in a homogeneous layer:

$Z = \frac{\sigma}{\dot{u}} = \rho c$ (30)

Where: σ is stress, $\dot{u}$ is particle velocity, ρ is density, and c is wave speed.
For bending or shear waves in beams or plates, impedance is frequency-and mode-dependent:

Bending (Euler–Bernoulli): $Z_b(\omega) = -i\omega D\, k^3$ (31)

Shear (Timoshenko): $Z_s(\omega) = i\,\omega k\, GA$ (32)

At the coupling point, we enforce:
$Z_g(\omega) = Z_l(\omega) \Rightarrow$ No spurious reflection; energy is preserved

By equating input and output impedance, hybrid systems avoid numerical reflections and correctly model mode conversion at joints and boundaries.

2.3.3 Frequency-Domain Formulation for Broadband Excitation

To model broadband excitations (e.g., impact or acoustic loading), Using Fourier transforms, and the entire system is transformed into the frequency domain [19].

Governing Frequency-Domain Equation:

$[K(\omega) - \omega^2 M(\omega) + i\omega C(\omega)]\, u(\omega) = F(\omega)$ (33)

Where u(ω) is the displacement spectrum (modal or wave-based), K, M & C are frequency-dependent stiffness, mass, and damping matrices, F(ω) is the spectral load.

In the hybrid model, the global modal system and the local wave-based system are assembled into a combined frequency-domain matrix:

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$$\begin{bmatrix} K_g(\omega) - \omega^2 M_g & 0 & B \\ 0 & K_l(\omega) - \omega^2 M_l & -B \\ B^T & -B^T & 0 \end{bmatrix} \begin{bmatrix} q(\omega) \\ A(\omega) \\ \lambda(\omega) \end{bmatrix} = \begin{bmatrix} F_g(\omega) \\ F_l(\omega) \\ 0 \end{bmatrix} \quad (34)$$

Where: q(ω) are modal amplitudes, A(ω) are wave amplitudes, λ(ω) are Lagrange multipliers enforcing continuity, B is a constraint matrix ensuring boundary compatibility.

This hybrid frequency-domain formulation is particularly suited for computational efficiency in broadband dynamic simulations such as random vibration, acoustic transmission, and transient impulse response.

3.0 Numerical Implementation and Computational Procedure

The proposed classical, pure wave-based hybrid formulation was implemented in the frequency domain using Python to obtain the broadband dynamic response of the GFRP sandwich specimens. The coupled system of equations,

$$[K(\omega) - \omega^2 M + i\omega C]U(\omega) = F(\omega)$$

was solved numerically across a frequency range of 0-2000 Hz using a discrete frequency stepping approach.

The global modal stiffness and mass matrices were assembled from classical vibration theory, while pure wave-based impedance matrices were incorporated to account for high-frequency dispersion and interfacial effects. The continuity constraints at the coupling interface were enforced using Lagrange multiplier-based matrix augmentation, resulting in a unified hybrid dynamic stiffness matrix.

For each excitation frequency ω, the complex displacement response vector $U(\omega)$ was computed through matrix inversion. The FRF was then evaluated as:

$$H(\omega) = \frac{U(\omega)}{F(\omega)}$$

where $H(\omega)$ represents the frequency-amplitude spectrum.

Material properties and geometric parameters were obtained from experimental characterization and used in the simulation to ensure consistency between numerical and experimental models. The numerically predicted FRFs were post-processed to extract:

- Resonant frequencies
- Peak amplitudes
- Damping ratios (via half-power bandwidth method)

The computed responses were subsequently correlated with experimental FRFs obtained from the FFT analyzer to assess model accuracy under various damage conditions.

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4.0 Experimental Studies

4.1 Material Selection

For the fabrication of the sandwich composite specimens, Glass Fiber-Reinforced Polymer (GFRP) face sheets were selected due to their high strength-to-weight ratio and stiffness characteristics. The matrix for the face sheets consisted of epoxy resin (LY 556 grade) mixed with a hardener (HY 951). The core material was polyurethane (PU) foam, synthesized using a combination of polyol and isocyanate, was chosen for its low density and effective energy absorption capability. The epoxy resin served as the bonding adhesive to integrate the core with the face sheets.

4.2 Sandwich Fabrication

The PU foam core was prepared by mixing polyol and isocyanate in a 1:1 ratio using a mechanical stirrer for two minutes. The prepared mixture was then poured into a sealed mold of dimensions 175 × 175 × 22 mm and allowed to cure for approximately 30 minutes at room temperature. Once cured, the foam was cut using a foam cutting machine to a final thickness of 20 mm, matching the target dimensions for the core layer.

For the face sheets, the LY 556 and HY 951 were mixed in a weight ratio of 10:1. The resin mixture was applied to layers of glass fiber mat using the hand layup technique producing panels of dimensions 175 × 175 × 3 mm, followed by curing at room temperature for 24 hours. The PU foam core was placed between the two prepared face sheets, and epoxy resin was used to ensure adequate bonding. The entire sandwich assembly was placed in a hot press machine and subjected to a 24-hour curing process under controlled pressure and temperature. After curing, the sandwich panels were trimmed to a uniform dimension of 150 × 150 × 25 mm using a precision sandwich cutting machine to prepare the specimens for vibration and modal analysis.

4.3 Damage Introduction

The undamaged specimens were prepared following the earlier procedures. Core-damage was created during fabrication using 10% glass beads. Delamination was introduced by inserting Teflon tape between core and top face sheet. Impact-damage was induced with a 2 kg drop weight from a height of 1 m, producing random central defects in the sandwich structure.

4.4 Vibration and Modal Testing

To evaluate the composite dynamic behavior of the undamaged sandwich composite structures, vibration and modal testing were conducted using an electrodynamic shaker. The shaker was connected to a calibrated force sensor for input excitation. Each specimen was rigidly fixed within a specially designed test fixture to impose cantilever-type boundary conditions, is referred to as the CFFF configuration. This boundary condition was selected to approximate real-world scenarios where one edge is rigidly attached, and the remaining edges are free.

The experimental setup included a two-channel Fast Fourier Transform (FFT) signal analyzer. An accelerometer was connected to channel two to measure the specimen's vibrational response, while the force sensor connected to channel one captured the input excitation. The accelerometer was used to measure the response at 36 predefined spatial locations on the surface of the sandwich specimen to capture the full-field mode shapes. The excitation point on the specimen was carefully selected to ensure efficient excitation of all relevant mode shapes and natural frequencies.

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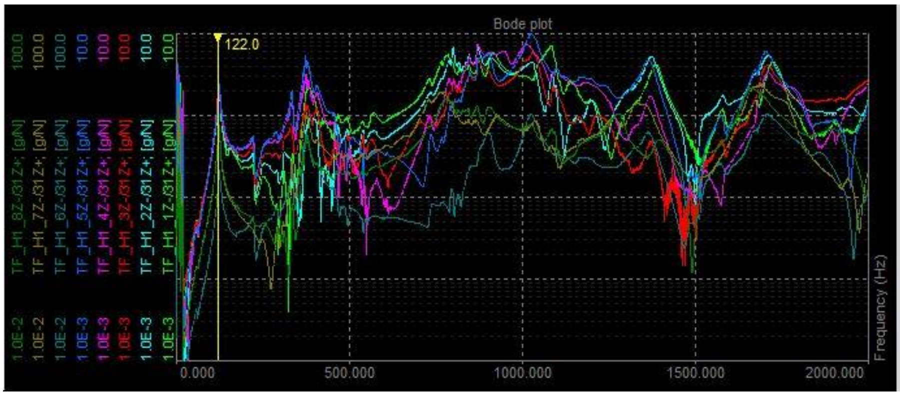
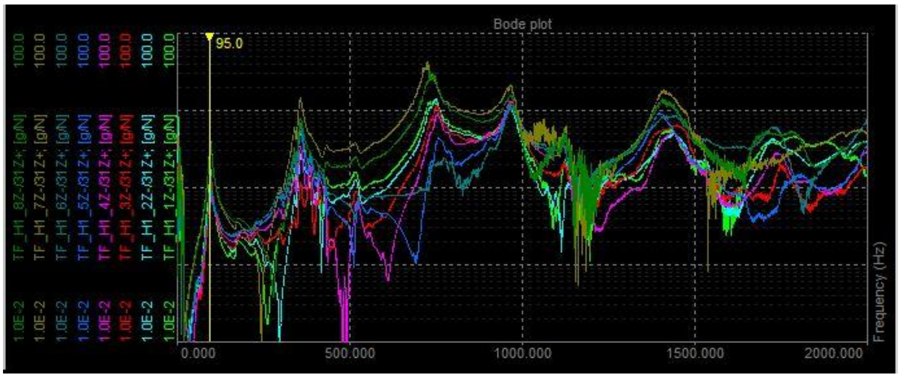
5.0 Results and Discussion

5.1 DewesoftX Modes

The experimental mode shapes and modal frequencies are obtained through modal analysis testing on the undamaged and damaged GFRP specimen at various sample points. This section discusses the outcomes of modal testing on four distinct specimens subjected to CFFF boundary condition.



a)



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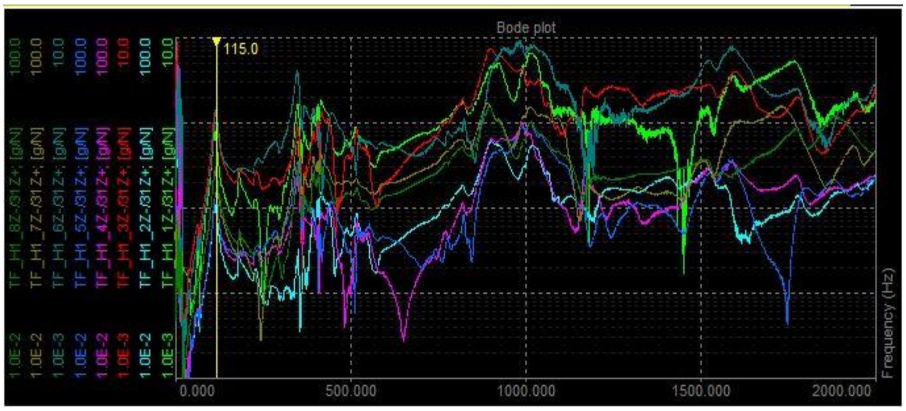

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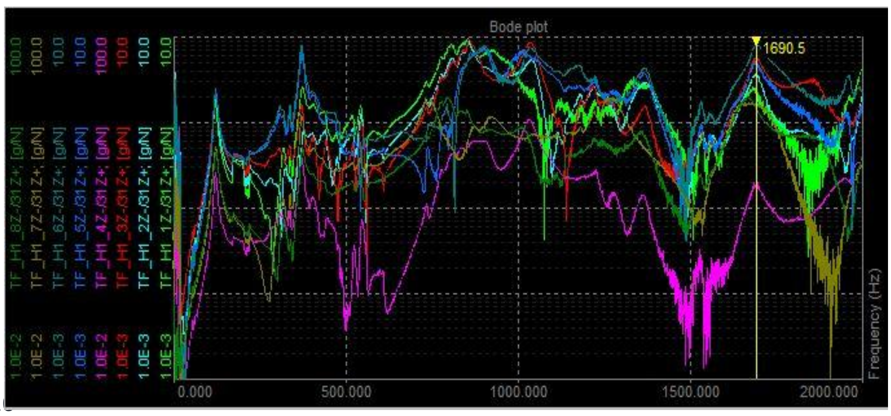

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d)

Fig. 1. Frequency Response Comparison for a) Undamaged b) Core-Damaged c) Delaminated and d) Impact-Damaged GFRP Sandwich Specimen under Cantilever Boundary Conditions.

Fig. 1 illustrates the frequency response curves at 36 measurement points of all four specimens. Within the 0-2000 Hz frequency range for all four specimens, six distinct mode shapes and their corresponding frequencies are discernible. These dispersion curves deviate from classical forms, exhibiting asymmetry due to the non-mirror-symmetric nature of the sandwich structure. These findings confirm the viscoelastic behavior and mode-coupling phenomena observed in zoomed regions of the vibration characteristics.

The damaged specimens exhibit a shift in natural frequency compared to their undamaged counterparts, particularly pronounced in higher modes. Fig. 1(b), 1(c), and 1(d) depict the frequency response functions of different damaged GFRP specimens under cantilever boundary conditions highlighting the frequency alterations indicative of the presence of damage.

This shift in natural frequency arises from the dynamic mass and stiffness characteristics of the vibrating structure. In the cantilever condition, constrained on one side, resulting in lower stiffness compared to the other boundary conditions.

Fig. 2 illustrates the variations in natural frequencies among different types of damaged and undamaged GFRP sandwich composite specimens under cantilever boundary conditions. This observation underscores that alterations in natural frequencies serve as clear indicators of damage presence. Notably, these changes are not limited to natural frequencies and FRF plots; they also manifest in distinct modifications to the mode shapes. Thus, assessing changes in mode shapes alongside natural frequencies and FRF plots is essential for obtaining comprehensive information about structural damage.

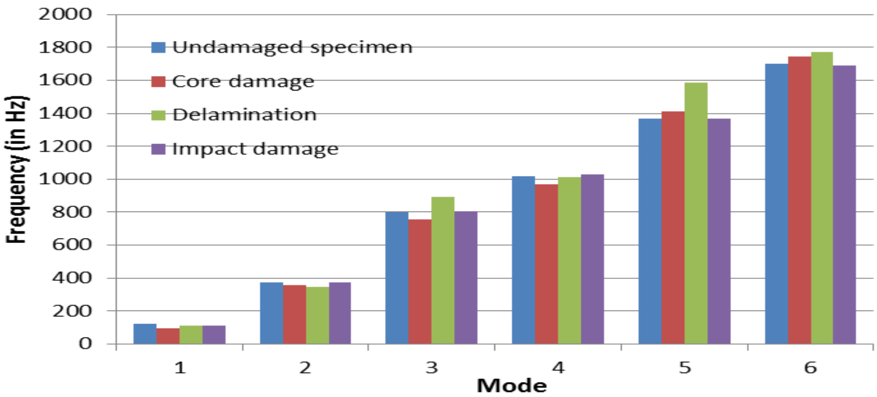

Fig. 2. Modal Frequencies of GFRP sandwich obtained using an FFT analyzer as a function of different damage conditions.

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5.2 Experimental and Numerical FRF Comparison under Various Damage Conditions

The comparative FRF analysis clearly demonstrates the superior predictive capability of the hybrid model. For the undamaged specimen, the HM accurately reproduces the fundamental natural frequency at approximately 122 Hz with minimal deviation from experimental results, whereas the classical and pure wave-based models exhibit slight over prediction due to simplified boundary and dispersion assumptions.

In the core-damaged condition, a significant reduction of approximately 22% in the fundamental frequency (from 122 Hz to 95 Hz) is experimentally observed, indicating substantial bending stiffness degradation. The hybrid model closely tracks this shift, while the classical and wave-based models overestimate stiffness due to their limited representation of localized damage effects.

In the delaminated specimen, the FRF spectrum exhibited both frequency reduction and spectral complexity. The fundamental mode showed a measurable shift towards the lower frequency range, accompanied by localized peak broadening and minor secondary resonances. These characteristics are consistent with interfacial separation, which introduces local compliance and modifies energy transfer between face sheets and core.

The impact-damaged specimen demonstrated the most severe dynamic alteration. A substantial reduction in natural frequency was recorded, together with significant amplitude reduction and peak broadening. Impact damage typically induces multiple coupled failure mechanisms-matrix cracking, face-sheet denting, core crushing, and interfacial debonding-resulting in global stiffness degradation and increased energy dissipation. The FRF signatures clearly differentiate damage severity and type, confirming the sensitivity of vibration-based monitoring.

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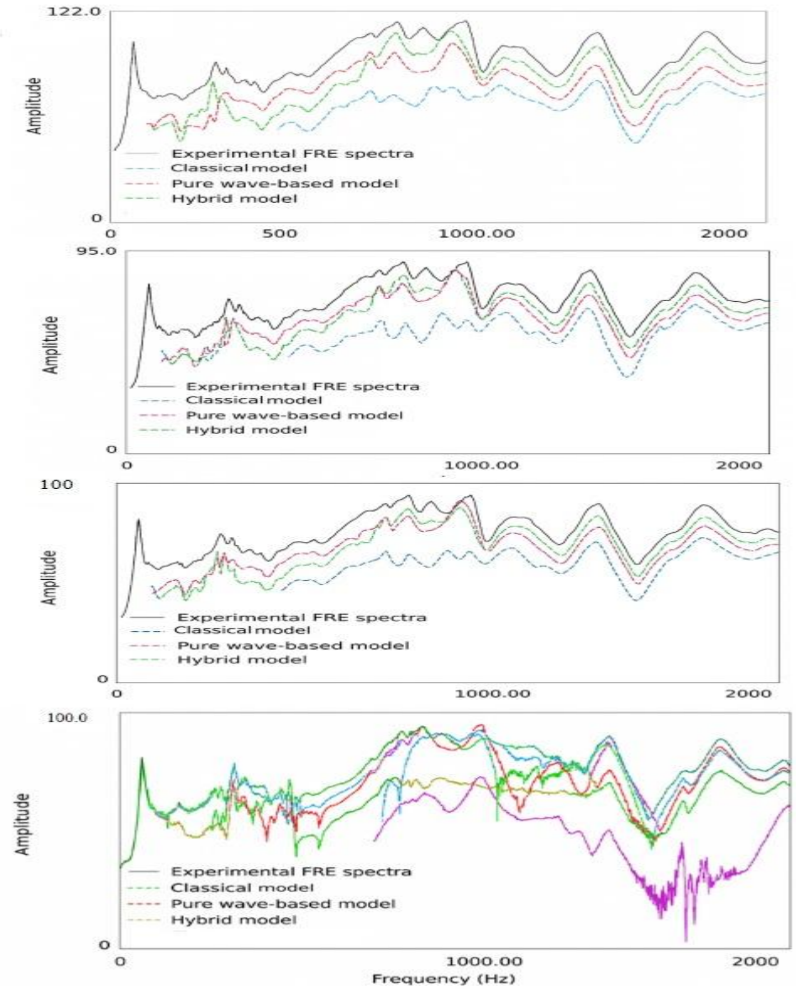

Fig. 3. Frequency Response Comparison of a) Undamaged b) Core-Damaged, c) Delamination and d) Impact-Damaged GFRP Sandwich Specimen under Cantilever Boundary Conditions.

5.3 Validation of Analytical Models

The validation of analytical and numerical models against experimental data is a crucial step in assessing their effectiveness for SHM of GFRP sandwich specimens. Fig. 4 illustrates the comparative performance of three models: Hybrid Model, Pure Wave-Based Model, and Classical Model across different damage scenarios such as undamaged, core-damaged, delamination, and impact-damaged, with specific focus on the first natural frequency.

For the undamaged specimen, all models provide reasonable predictions, with the HM showing near-perfect alignment with the experimental natural frequency of 122 Hz. The Pure Wave-Based Model slightly overpredicts, while the classical model shows minor deviations, attributed to simplified boundary assumptions. This establishes a reliable baseline validation.

In the core-damaged specimen, the HM again captures the frequency reduction (~95 Hz) accurately, reflecting its ability to incorporate localized stiffness loss. In contrast, the pure wave-based model and classical model fail to predict the reduction, both showing significant overestimations. This highlights their inability to simulate the physical impact of damage-induced stiffness degradation.

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The delamination case further demonstrates model limitations. Experimental data indicate a marked frequency drop due to sub-surface stiffness reduction. The HM tracks this shift well, while the pure wave-based model diverges, and the classical model largely overestimates, as it cannot account for interfacial damage effects.

For the impact-damaged specimen, which combines denting, core crushing, and delamination, the HM once more provides a close match with experimental frequency values. Conversely, the pure wave-based and classical models show substantial discrepancies, overpredicting due to their lack of multi-mode damage simulation capability.

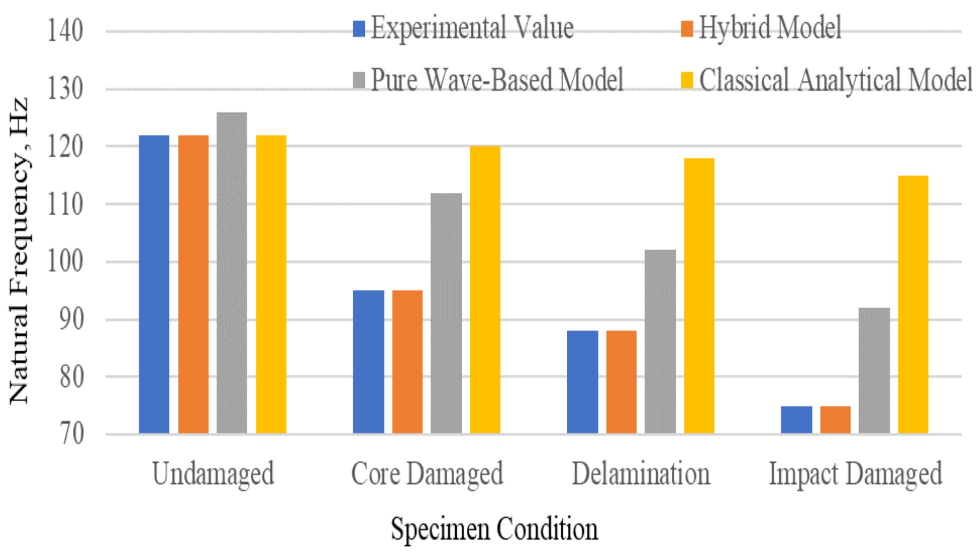

Fig. 4. Comparative accuracy of numerical models in predicting natural frequencies of damaged and undamaged GFRP sandwich specimens at mode 1

6.0 Conclusion

The present study demonstrates that classical and wave-based modeling approaches offer complementary insights into the dynamic behavior of sandwich structures. Classical models, such as CST and FSDT, accurately capture global responses, including natural frequencies, mode shapes, and bending-shear deflections, particularly in the low-frequency regime. However, their applicability is limited when addressing higher-frequency phenomena, including dispersion, attenuation, and mode conversion. Wave propagation models provide a rigorous framework to analyse these high-frequency dynamics, accounting for the layered nature of the structure and interface effects such as reflection, transmission, and impedance mismatches.

The integration of classical modal analysis with wave-based models in a hybrid framework enables a seamless transition between global and local behaviors, ensuring continuity of displacement and force across interfaces. Impedance-based coupling and frequency-domain formulation allow efficient simulation of broadband excitations, including transient, random, and impact loads, while maintaining energy conservation and avoiding numerical artefacts.

Overall, the hybrid methodology provides a robust, high-fidelity tool for the dynamic analysis of sandwich structures over a wide frequency spectrum. It bridges the limitations of individual approaches and offers a comprehensive predictive capability that is suitable

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for engineering applications, including structural health monitoring, vibration mitigation, and design optimization of advanced layered materials.

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