ISSN: 1672 - 6553

JOURNAL OF DYNAMICS AND CONTROL

VOLUME 10 ISSUE 01: P130 - 152

INFLUENCE OF SORET–DUFOUR EFFECTS ON NONLINEAR CONVECTION OF JEFFREY FLUID IN A STRETCHABLE CHANNEL WITH LORENTZ AND RADIATION FORCES

R. D. Jagadeesha[1], M. S. Sunitha[2], S. Kemparaju[3], P. B. Sampath Kumar[4,5]

[1] Department of Mathematics, Government First Grade College for Women, Holenarasipura, Karnataka, India

[2] Department of Mathematics, Government First Grade College, Tumkur, Karnataka, India

[3] Department of Mathematics, Government First Grade College, Ramanagara, Karnataka, India

[4] Department of Studies and Research in Mathematics, Ramanagara PG Centre, Bangalore University, Karnataka 562159, India

[5] Universidad Bernardo O'Higgins, Facultad de Ingeniería, Ciencia y Tecnología, Departamento de Formación y Desarrollo Científico en Ingeniería, Av. Viel 1497, Santiago, Chile

https://doi.org/10.71058/jodac.v10i01008

INFLUENCE OF SORET–DUFOUR EFFECTS ON NONLINEAR CONVECTION OF JEFFREY FLUID IN A STRETCHABLE CHANNEL WITH LORENTZ AND RADIATION FORCES

R. D. Jagadeesha[1], M. S. Sunitha[2], S. Kemparaju[3], P. B. Sampath Kumar[4,5]

[1] Department of Mathematics, Government First Grade College for Women, Holenarasipura, Karnataka, India
[2] Department of Mathematics, Government First Grade College, Tumkur, Karnataka, India
[3] Department of Mathematics, Government First Grade College, Ramanagara, Karnataka, India
[4] Department of Studies and Research in Mathematics, Ramanagara PG Centre, Bangalore University, Karnataka 562159, India
[5] Universidad Bernardo O'Higgins, Facultad de Ingeniería, Ciencia y Tecnología, Departamento de Formación y Desarrollo Científico en Ingeniería, Av. Viel 1497, Santiago, Chile
jagdeesh25gfgcwhnp@gmail.com, ms.sunitha0204@gmail.com, skemparaju74@gmail.com, sampathkumarpb123@gmail.com
***Corresponding Author: R. D. Jagadeesha**

Abstract: This work investigates the MHD nonlinear convection flow of a Jeffrey fluid over a vertical surface, considering cross-diffusion, nonlinear radiation, heat generation, thermophoresis, and convective boundary conditions. The governing PDEs are reduced to nonlinear ODEs using similarity transformations and solved via the RKF-45 method. Validation against published results shows excellent agreement. Parametric analysis reveals that the Deborah number increases velocity and boundary layer thickness, while the Dufour and Soret effects enhance temperature and concentration profiles, respectively. The findings provide useful insights into heat and mass transfer in non-Newtonian MHD flows relevant to engineering applications.

Keywords: Jeffrey fluid; Convection; Thermal radiation; Cross-diffusion; Heat generation.

Introduction

Soret and Dufour effects (also referred to as thermal diffusion and diffusion-thermo phenomena, respectively) are two closely related mechanisms in fluid dynamics that describe the coupling between temperature and concentration gradients. The Soret effect occurs when a temperature gradient within a fluid mixture drives mass diffusion, often causing separation of components toward the lower temperature region. Conversely, the Dufour effect refers to the

https://doi.org/10.71058/jodac.v10i01008

generation of an additional heat flux induced by concentration gradients, which modifies the overall energy transport in the system. These cross-diffusion effects are significant in diverse fields such as chemical engineering, geophysics, and astrophysics. Several studies have explored their implications in various flow configurations. Turkyilmazoglu and Pop [1] analysed Soret and Dufour characteristics in viscous flow past a plate under the influence of a Lorentz force. Zheng et al. [2] studied their influence on hydromagnetic viscous flow over oscillatory stretching surfaces. Reddy and Chamkha [3] examined Soret and Dufour effects in magnetized nanofluid flows, while. Sampath Kumar et al. [4] investigated the nonlinear thermal convection effects on heat and mass transfer in a dissipative Jeffrey fluid, incorporating the influence of cross-diffusion and convective boundary conditions. In addition, Hayat et al. [5] considered 3D radiative flow incorporating cross-diffusion, and Zia et al. [6] analyzed the combined impact of Soret and Dufour on Casson fluid with heat generation and thermal radiation. Madan Kumar et al. [7] formulated a mathematical model describing the two-dimensional steady incompressible flow of Jeffrey fluid past a vertical stretching sheet embedded in a Darcy porous medium. The model including Soret and Dufour effects. Subhan Ullah et al. [8] investigated hydro-magnetic Jeffery–Hamel flow in convergent/divergent channels with stretchable walls, considering the influences of Soret/Dufour effects, Joule heating, chemical reaction, and heat source. Their results revealed that heat source and Dufour effects significantly enhance the temperature distribution, whereas the Soret number leads to a reduction in concentration. Several researchers have extensively examined cross-diffusion effects owing to their wide-ranging and promising applications Ramana Reddy et al. [9], Revathi et al. [10], Saima Noreen et al. [11], Raju and Sandeep [12], eid et al. [13] , Siddique et al. [14] and Ullah et al. [15].

The study of natural, forced, and mixed convection flows of working fluids has drawn considerable attention because of its broad engineering relevance, with applications spanning nuclear reactor cooling, solar energy collection, thermal energy storage, electronic device cooling, food preservation, and cryogenic systems. Within this framework, Gorla and Sidawi [16] examined free convection flow induced by a stretching elastic plate under the influence of transpiration cooling. Wang [17] addressed mixed convection heat transfer of non-Newtonian fluids along a vertical surface, while Chamkha [18] investigated hydromagnetic natural convection over a linearly stretching plate. Rashidi et al. [19] explored mixed convection boundary-layer flow of micropolar fluids on heated vertical sheets, presenting analytical solutions through the homotopy analysis method. Giresha et al. [20] studied the mixed convection flow of a Maxwell fluid containing suspended particles, incorporating the effects of non-linear thermal radiation and a non-uniform heat source or sink. A vast body of literature exists on convection flows involving various fluids under diverse conditions [21–24]. However, most of these studies are confined to linear thermal convection, which assumes a small temperature difference between the heated surface and the surrounding fluid. In practical applications such as solar collectors, electronic cooling systems, nuclear reactors, and energy storage devices, nonlinear density–temperature variations in the buoyancy force significantly influence flow, heat, and mass transfer characteristics. Recognizing this, researchers including Vajravelu et al. [25], Kameswaran et al. [26], and Sachin et al. [27] have conducted comprehensive investigations into the effects of nonlinear convection. Recently, Jyoti et al. [28] investigated nonlinear convection and radiative heat transfer in kerosene–alumina nanofluid flow between two parallel plates, accounting for variable viscosity effects. Their study is particularly relevant for engineering applications involving enhanced heat transfer performance. Magammad et al. [29] examined buoyancy-driven nonlinear combined convection and thermal radiative flow of a Newtonian fluid (TRNFF) over a nonlinearly stretching vertical sheet.

https://doi.org/10.71058/jodac.v10i01008

Non-Newtonian fluid flows, particularly Jeffrey and other viscoelastic fluids, are of considerable interest in engineering applications such as polymer processing, biomedical devices, and heat exchangers. Several studies, including those by Hayat et al. [30], Dalir [31], Qasim [32], Shehzad et al. [33], Ramesh [34], and Veera Krishna and Chamkha [35], have investigated Jeffrey fluid flows under various conditions. However, most research on MHD flows focuses on Newtonian fluids or linear thermal effects, while practical systems often involve nonlinear thermal radiation, cross-diffusion (Dufour and Soret effects), thermophoresis, heat generation, and convective boundaries, which significantly influence heat and mass transfer. The combined effects of MHD, nonlinear convection, cross-diffusion, and thermophoretic forces in Jeffrey fluid flows over vertical surfaces remain largely unexplored. This study addresses this gap through a parametric analysis of Jeffrey fluid MHD flows, providing insights for optimizing industrial and engineering processes involving complex fluids.

Mathematical Formulation

We examine a two-dimensional boundary layer flow of Jeffrey fluid over a linearly stretching surface situated at $y = 0$, with the flow domain extending to $y > 0$. The sheet is stretched by equal and opposite forces applied along the x-axis, producing a velocity distribution $U_w = ax$, where $a > 0$ is a constant. The coordinate system is defined such that the x-axis lies along the stretching surface while the y-axis is normal to it. Thermal and solutal boundary conditions are prescribed by assuming that the surface is heated and enriched with concentration values T_f and C_f, associated with respective heat and mass transfer coefficients h_1 and h_2. Far from the surface, the fluid attains constant ambient conditions, namely temperature T_∞ and concentration C_∞. This configuration provides a framework to analyse coupled heat and mass transfer in Jeffrey fluids influenced by stretching sheet dynamics.

 https://doi.org/10.71058/jodac.v10i01008

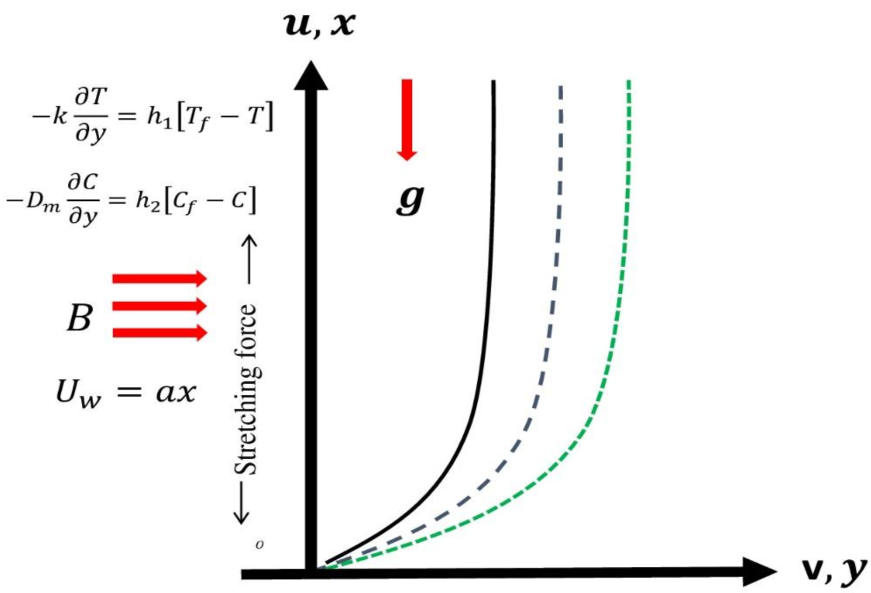

Figure 1: Geometry of the problem.

The problem consideration is governed by the following equations (see Ahmed et al. [7], Sachin et al. [19, 22]);

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\nu}{1+\lambda_1}\right)\left[\frac{\partial^2 u}{\partial y^2} + \lambda_2\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right)\right]$$
$$-\frac{\sigma B^2}{\rho}u + g(\beta_0(T - T_\infty) + \beta_1(T - T_\infty)^2 + \beta_2(C - C_\infty)), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p}(T - T_\infty)$$
$$+\frac{\mu}{\rho C_p(1+\lambda_1)}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \lambda_2\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)\right], \tag{3}$$

$$\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - \frac{\partial(V_T C)}{\partial y}, \tag{4}$$

The radiative heat flux expression in equation (1.1.3) is given by the Rosseland approximation as;

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial y}, \tag{5}$$

Where σ^* and k^* are the Stefan-Boltzman constant and the mean absorption coefficient correspondingly, and in view to equation (1.2.5) in equation (1.2.3) reduces to;

https://doi.org/10.71058/jodac.v10i01008

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}+\frac{16\sigma^*}{3\rho c_p k^*}\left[T^3\frac{\partial^2 T}{\partial y^2}+3T^2\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{Q_0}{\rho c_p}(T-T_\infty)$$

$$+\frac{\mu}{\rho C_p(1+\lambda_1)}\left[\left(\frac{\partial u}{\partial y}\right)^2+\lambda_2\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)\right], \quad (6)$$

The corresponding boundary conditions at the surface and far away from the surface are written as follows;

$$u=U_w(x), v=-V_w(x),\ -k\frac{\partial T}{\partial y}=h_1(T_f-T), -D_m\frac{\partial C}{\partial y}=h_2(C_f-C) \text{ at } y=0,$$

$$u\to 0,\quad \frac{\partial u}{\partial y}\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty, \quad (7)$$

In which u and v are the velocity components alongside the x- and y-axes respectively. where ν -dynamic viscosity, λ_1-ratio of relaxation and retardation time, λ_2 -retardation time, g-acceleration due to gravity, B -magnetic field, T_m -fluid mean temperature, β_2 -volumetric solute expansion coefficient, β_0 and β_1-linear and non-linear volumetric thermal expansion coefficient respectively, α_m -thermal diffusivity, D_m -solutal diffusivity coefficient, ρ -density, K_T -thermal diffusion ratio, C_s -concentration susceptibility, C_p -specific heat capacity, μ-dynamic viscosity, Q_0-dimensional heat generation/absorption coefficient, V_T -thermophoretic velocity, k -thermal conductivity, T -temperature of the fluid, C -concentration of the fluid, T_∞ -ambient temperature and C_∞- ambient concentration. The term V_T in equation (2.4) can be defined as follows;

$$V_T=-k_1\frac{\nu}{T_r}\frac{\partial T}{\partial y}, \quad (8)$$

here k_1 - thermophoretic coefficient and T_r -reference temperature.

Now, introduce the following similarity transformations

$$\eta=\sqrt{\frac{U_w}{\nu x}}y,\ u=axf'(\eta),\ v=-\sqrt{a\nu}f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},$$

$$\phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty},\ T=T_\infty(1+(\theta_w-1)\theta(\eta)) \text{ with } \theta_w=\frac{T_f}{T_\infty}, \theta_w>1 \quad (9)$$

Into the equations (1.2.2) to (1.2.7), we get

https://doi.org/10.71058/jodac.v10i01008

$$f''' + (1+\lambda_1)(ff'' - f'^2) + \beta(f''^2 - ff'''')$$
$$+\lambda(1+\lambda_1)\{(\theta + \alpha\theta^2 + N\phi)\} - (1+\lambda_1)Mf' = 0, \quad (10)$$

$$(1+\lambda_1)\left\{\theta'' + R\left[\left(1+(\theta_w-1)\theta(\eta)\right)^3\theta''(\eta) + 3(\theta_w-1)\theta'^2(\eta)\left(1+(\theta_w-1)\theta(\eta)\right)^2\right] + \Pr\left(f\theta' + D_f\phi'' + Q\theta\right)\right\} + PrEc\{f''^2 + \beta f''(f'f'' - ff''')\} = 0, \quad (11)$$

$$\phi'' + Scf\phi' + ScSr\theta'' - Sc\tau(\phi'\theta' + \phi\theta'') = 0, \quad (12)$$

and the corresponding boundary conditions become;

$$f' = 1,\quad f = S,\quad \theta' = -Bi_1(1-\theta),\quad \phi' = -Bi_2(1-\phi) \quad at\ \eta = 0,$$
$$f' \to 0,\quad f'' \to 0,\quad \theta \to 0,\quad \phi \to 0 \quad as\ \eta \to \infty. \quad (13)$$

The dimensionless mixed convection parameter, local Grashof number, local Reynolds number, Deborah number, nonlinear convection parameter, buoyancy ratio parameter, Magnetic parameter, Dufour number, Prandtl number, heat generation parameter, radiation parameter, temperature ratio parameter, Eckert number, Schmidt number, thermophoretic parameter, Soret number, suction parameter, thermal Biot number, and concentration Biot number are;

$$\lambda = \frac{Gr_x}{(Re_x)^2},\ Gr_x = \frac{g\beta_0(T_f - T_\infty)U_w^3}{\nu^2 a^3},\ Re_x = \frac{U_w^2}{\nu a},\ \beta = \lambda_2 a,\ \alpha = \frac{\beta_1(T_f - T_\infty)}{\beta_0},\ N = \frac{\beta_2(C_f - C_\infty)}{\beta_0(T_f - T_\infty)},$$

$$M = \frac{\sigma B^2}{\rho a},\ D_f = \frac{D_m K_T(C_f - C_\infty)}{Cs\, C_p \nu(T_f - T_\infty)},\ Pr = \frac{\nu}{\alpha_m},\ Q = \frac{Q_0}{a\rho c_p},\ R = \frac{16\sigma^* T_\infty^3}{3k^* k},\ \theta_w = \frac{T_f}{T_\infty},\ Ec = \frac{U_w^2}{C_p(T_f - T_\infty)},$$

$$Sc = \frac{\nu}{D_m},\ \tau = -\frac{k_1(T_f - T_\infty)}{T_r},\ Sr = \frac{D_m K_T(T_f - T_\infty)}{T_m \nu(C_f - C_\infty)},\ S = \frac{V_w}{\sqrt{a\nu}},\ Bi_1 = \frac{h_1}{k}\sqrt{\frac{\nu}{a}}\ Bi_2 = \frac{h_2}{D_m}\sqrt{\frac{\nu}{a}}\text{-}$$

The Skin friction coefficient, Nusselt number and Sherwood numbers are;

$$C_f = \frac{\tau_w}{\rho U_w^2},\ Nu_x = \frac{xq_w}{k(T_f - T_\infty)},\ Sh_x = \frac{xq_m}{D_m(C_f - C_\infty)} \quad (14)$$

By Fourier's law, τ_w -surface shear stress, q_w-surface heat flux and q_m -surface mass flux are given by;

$$\tau_w = \frac{\mu}{1+\lambda_1}\left[\mu\frac{\partial u}{\partial y} + \lambda_2\left(u\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^2 u}{\partial y^2}\right)\right]_{y=0},$$

$$q_w = -k\left(\frac{\partial T}{\partial y} + q_r\right)_{y=0},$$

$$q_m = -D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}. \quad (15)$$

https://doi.org/10.71058/jodac.v10i01008

Now by combining equation (1.2.9) and (1.2.15) in view of equation (1.2.14), we have obtained;

$$C_f Re_x^{1/2} = \frac{1}{1+\lambda_1}\left(f''(0) + \beta\left(f'(0)f''(0) - f(0)f'''(0)\right)\right),$$
$$Nu_x Re_x^{-1/2} = -(1 + R\theta_w^3)\theta'(0),$$
$$Sh_x Re_x^{-1/2} = -\phi'(0), \quad (16)$$

Physical Interpretation

The present study investigates the MHD nonlinear convection flow of Jeffrey fluid over a vertical surface, incorporating the combined influences of radiative heat transfer, cross-diffusion, thermophoresis, internal heat generation, and convective boundary conditions. A detailed parametric analysis is carried out by considering the effects of several governing parameters, including the magnetic parameter (M), Deborah number (β), mixed convection parameter (λ), relaxation-to-retardation ratio (λ_1), nonlinear convection parameter (α), buoyancy ratio parameter (N), suction/injection parameter (S), Dufour number (D_f), Soret number (Sr), Eckert number (Ec), heat generation parameter (Q), radiation parameter (R), temperature ratio parameter (θ_w), Prandtl number (Pr), thermophoretic parameter (τ), Schmidt number (Sc), thermal Biot number (Bi_1), and solutal Biot number (Bi_2). To evaluate the reliability of the adopted numerical method, the computed values of $f''(0)$ are compared with earlier results reported by Dalir [6] using IKBM and by Hayat et al. [5] employing HAM for a limiting case. The comparison demonstrates excellent agreement, as summarized in Table 1, confirming the accuracy of the present approach.

The analysis of heat transfer characteristics, illustrated in Figure 2, reveals that the Nusselt number exhibits an increasing trend with respect to the nonlinear convection parameter (α) and the mixed convection parameter (λ), while it shows a decreasing response to the Deborah number (β). Physically, this behavior is linked to the role of nonlinear convection, which amplifies the wall temperature gradient and thereby enhances the rate of heat transfer

https://doi.org/10.71058/jodac.v10i01008

from the surface to the fluid. In contrast, larger Deborah numbers correspond to stronger fluid elasticity and memory effects, which suppress thermal diffusion and consequently reduce heat transfer.

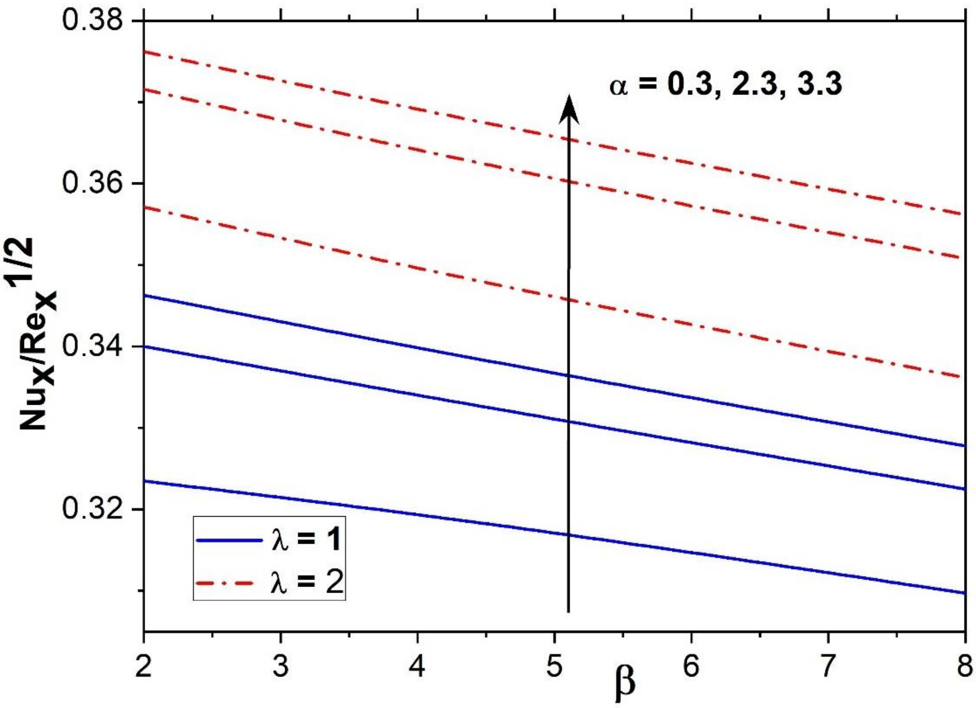

Figure 2: Influence of convection and Deborah number on Nusselt number.

Similarly, the mass transfer behavior shown in Figure 3 indicates that the Sherwood number rises steadily with increasing values of α, β, and λ. This suggests that nonlinear convection, higher Deborah numbers, and mixed convection collectively accelerate the transport of mass species, thereby thickening the concentration boundary layer and improving overall diffusion. Such observations highlight the significant influence of these parameters on controlling thermal and solutal transport in non-Newtonian MHD flows.

https://doi.org/10.71058/jodac.v10i01008

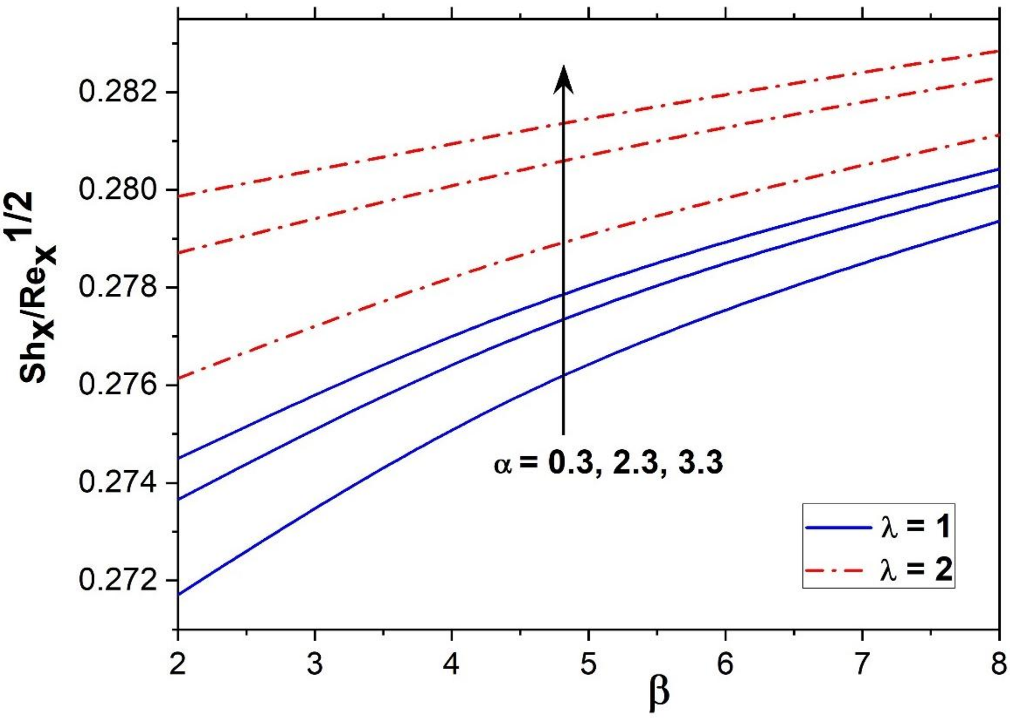

Figure 3: Influence of convection and Deborah number on Sherwood number.

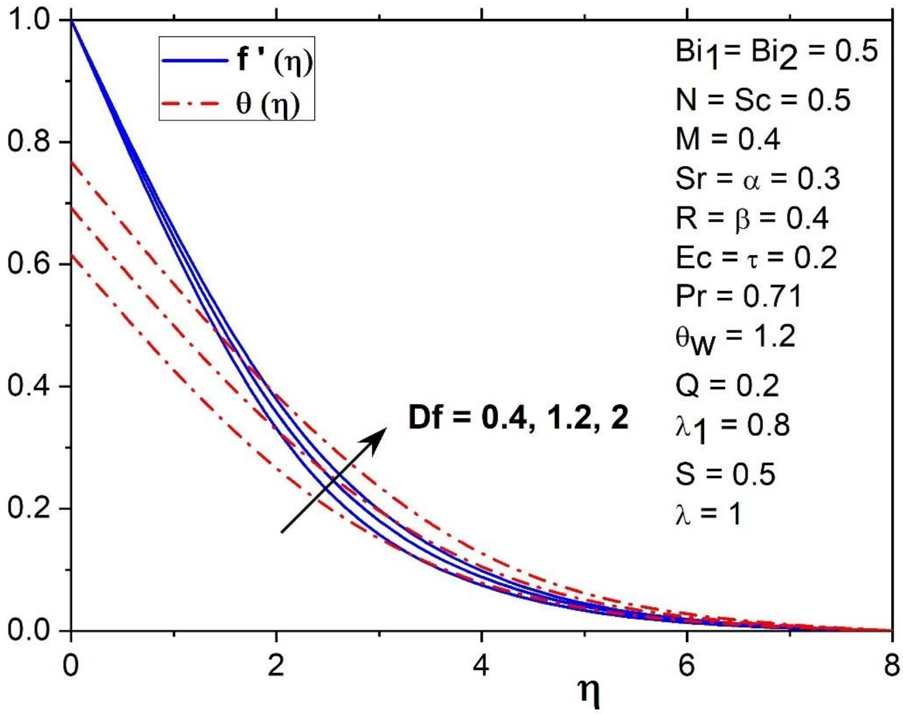

Figure 4: Influence of Dufour number on velocity and temperature profile.

https://doi.org/10.71058/jodac.v10i01008

The coupled influence of the Dufour number on velocity and temperature fields is highlighted in Figure 4. Both quantities increase with greater Dufour values, as this parameter represents the generation of heat flux induced by concentration gradients. Figure 5 illustrates that the Soret number enhances velocity and concentration distributions, owing to stronger diffusion effects. Figure 6 demonstrates the influence of the Deborah number on velocity behavior. As β increases, the velocity profile is significantly enhanced. Physically, a larger Deborah number indicates a higher retardation time, which reduces the effective viscosity and consequently accelerates the fluid within the boundary layer. The figure also reveals that the momentum boundary layer is thicker when mixed convection is present (λ = 1) compared to its absence. The velocity distribution under the influence of the magnetic parameter M, both with mixed convection (λ = 1) and without it (λ = 0), is illustrated in Figure 7. It can be noted that stronger magnetic effects suppress fluid motion, a result of the Lorentz force acting against the flow.

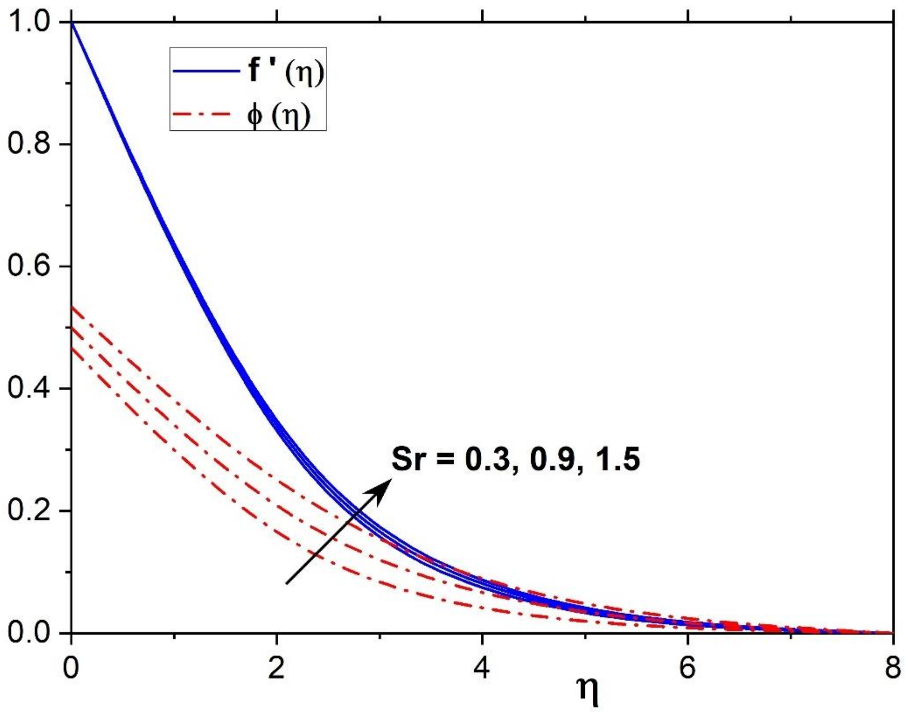

Figure 5: Influence of Soret number on velocity and concentration profile.

https://doi.org/10.71058/jodac.v10i01008

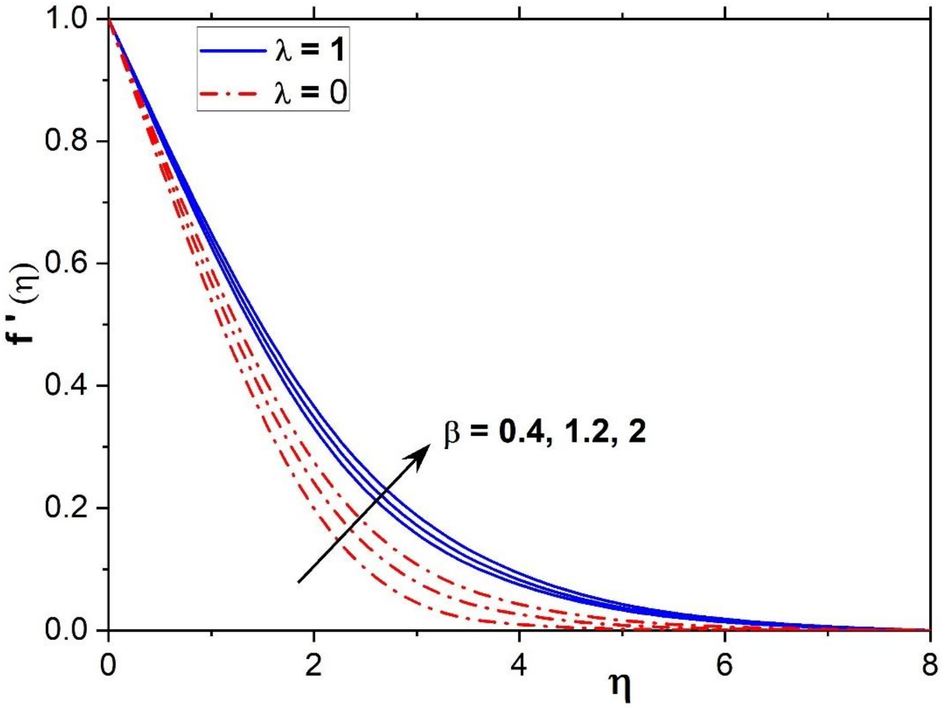

Figure 6: Influence of Deborah number on velocity profile.

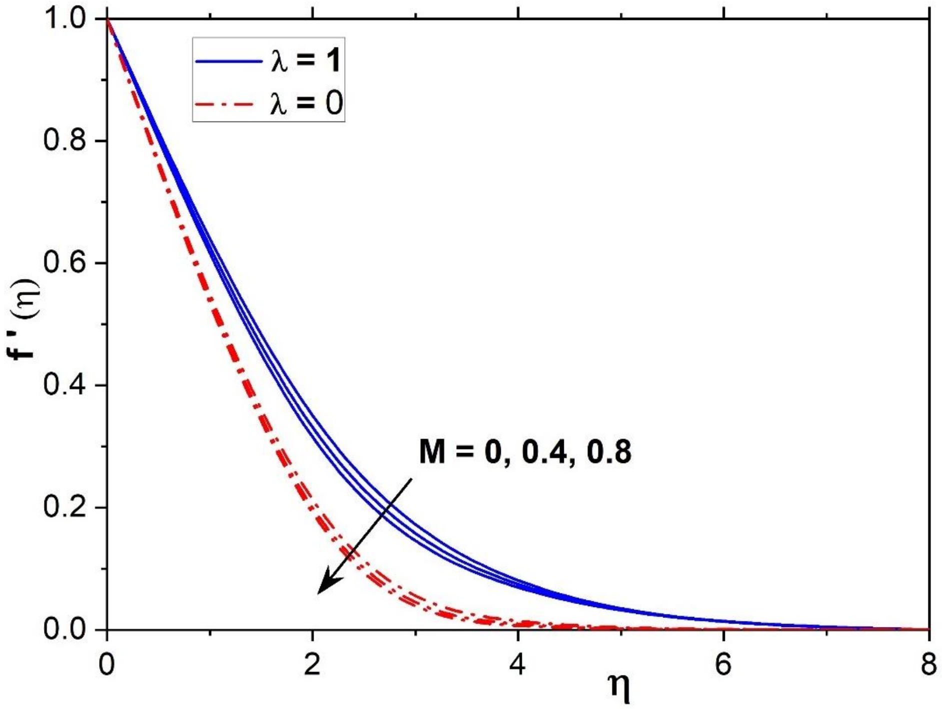

Figure 7: Influence of magnetic field on velocity profile.

 https://doi.org/10.71058/jodac.v10i01008

Radiation effects R are illustrated in Figure 8, where stronger radiative contributions are observed to expand the thermal boundary layer. This occurs because higher radiative heat flux transfers more energy from the surface into the fluid, thereby raising the fluid temperature and thickening the boundary layer. Similarly, Figure 9 demonstrates that the temperature profile increases with the temperature ratio parameter (θ_w), as a higher wall-to-fluid temperature ratio magnifies the thermal gradient near the surface, leading to enhanced heat transfer. The influence of the heat source parameter (Q) is highlighted in Figure 10, showing that internal heat generation contributes additional energy to the system, which simultaneously accelerates the fluid motion and elevates the temperature distribution within the boundary layer.

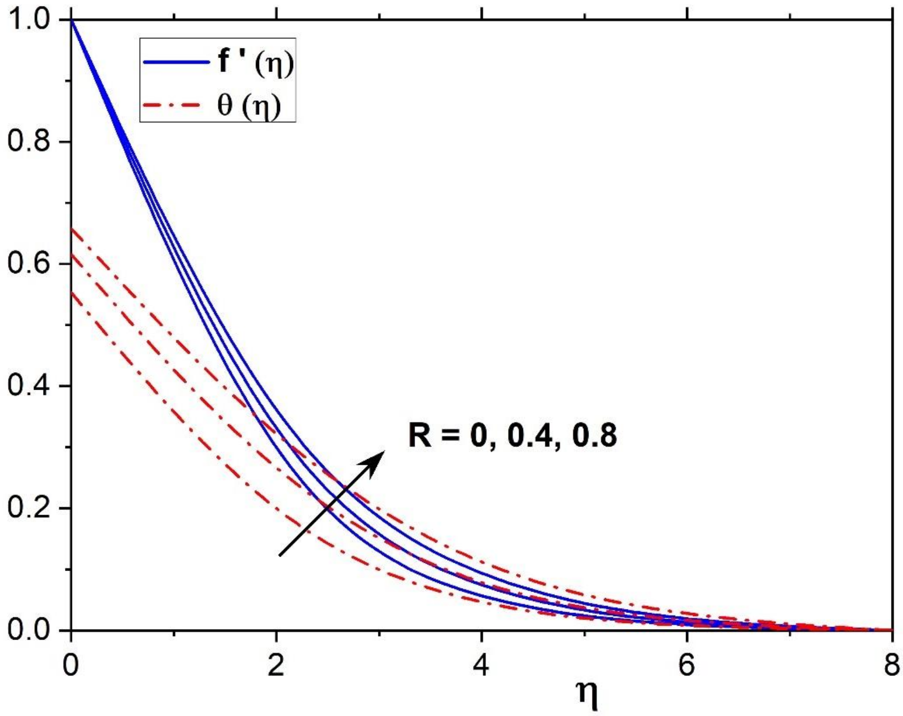

Figure 8: Influence of thermal radiation on temperature profile.

https://doi.org/10.71058/jodac.v10i01008

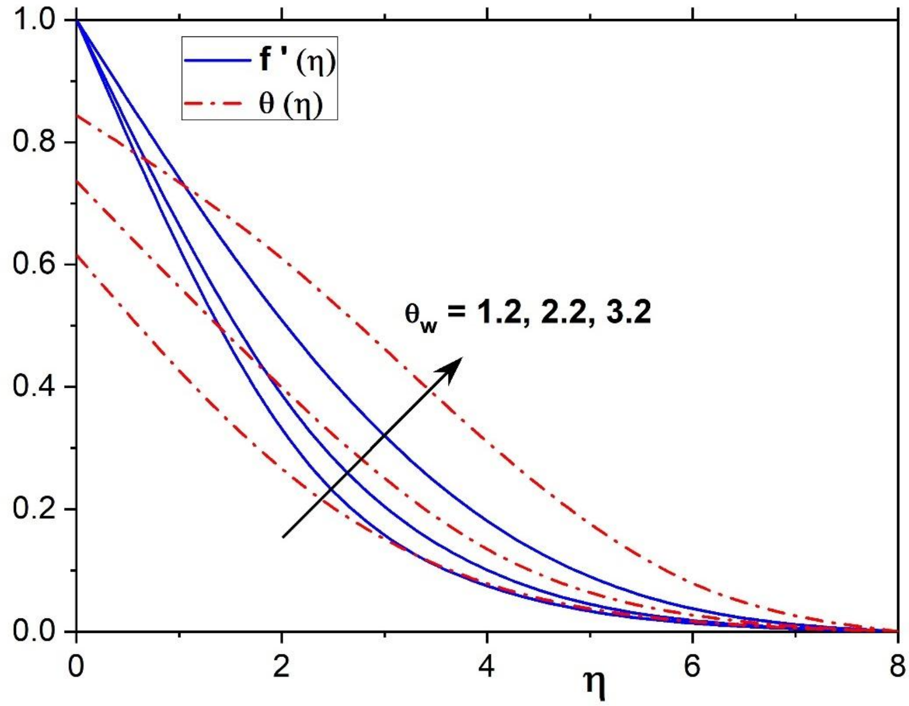

Figure 9: Influence of temperature ratio parameter on temperature profile.

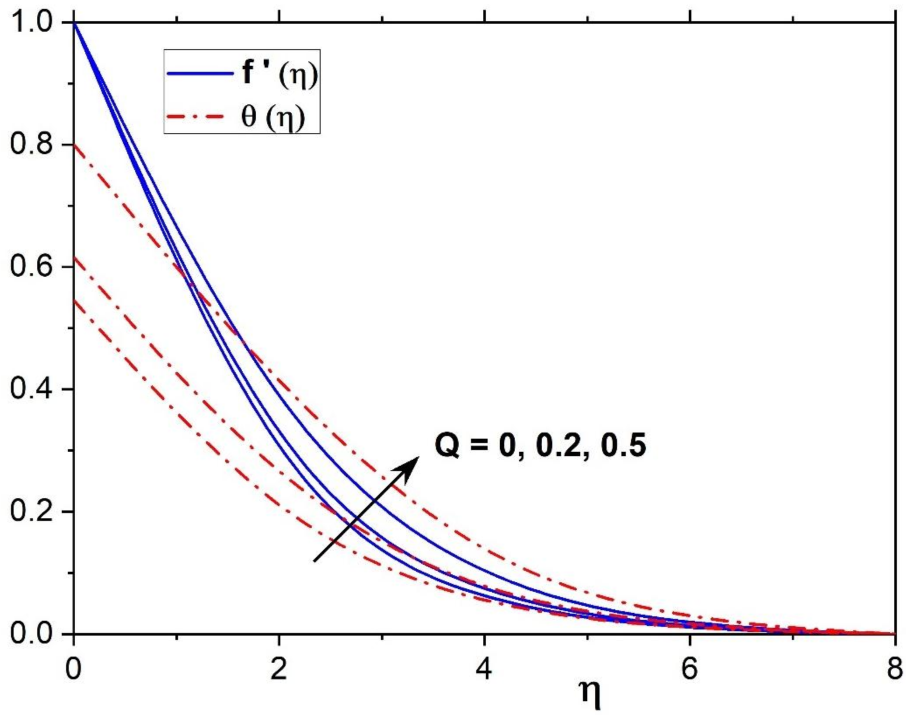

Figure 10: Influence of heat source parameter on temperature profile.

https://doi.org/10.71058/jodac.v10i01008

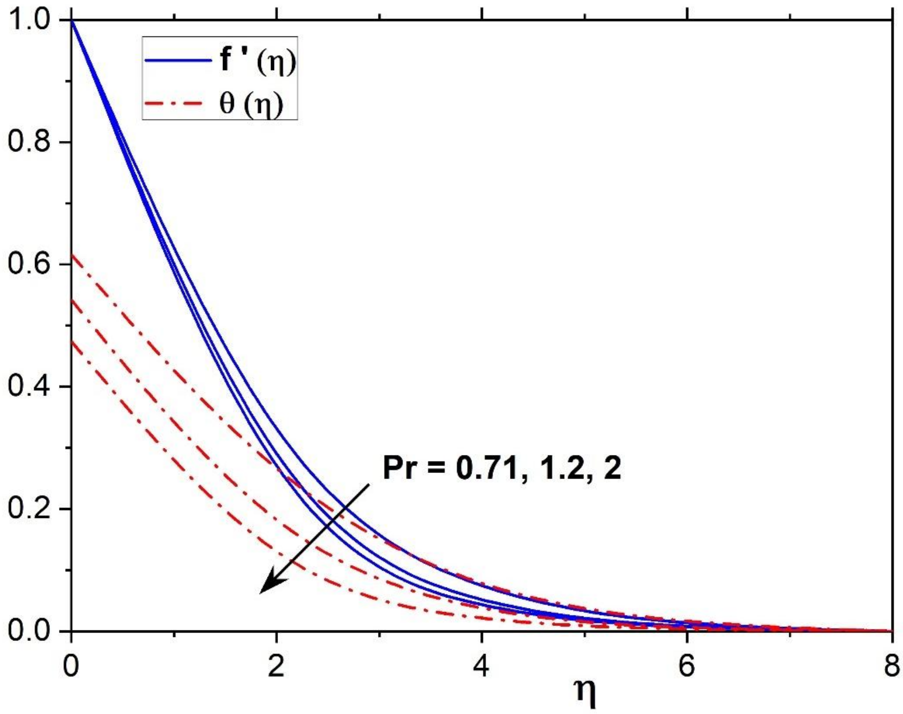

Figure 11: Influence of Prandtl number on temperature profile.

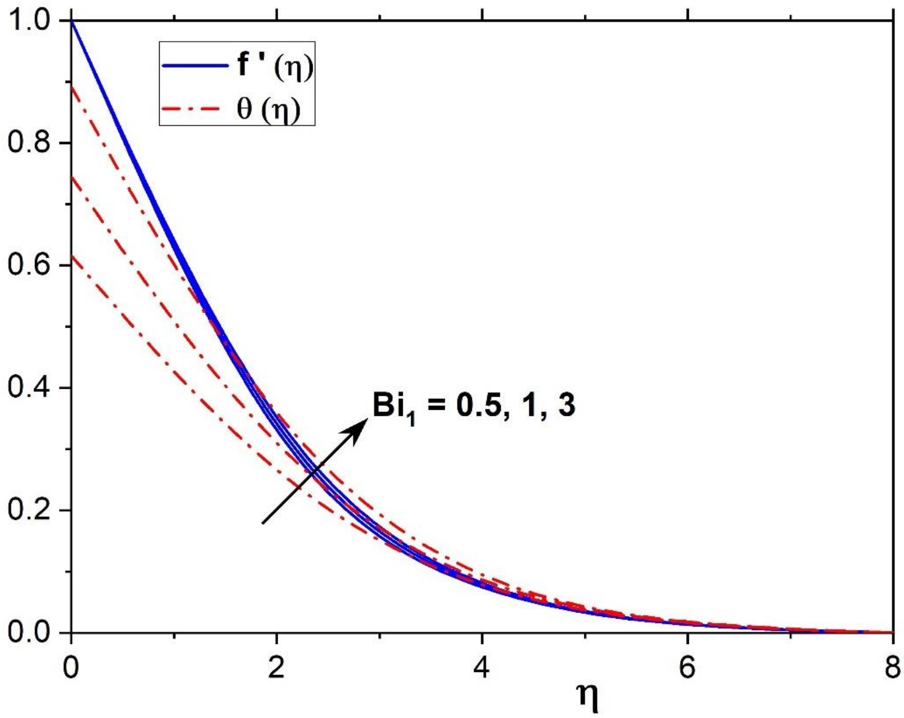

Figure 12: Influence of thermal Biot number on temperature profile.

 https://doi.org/10.71058/jodac.v10i01008

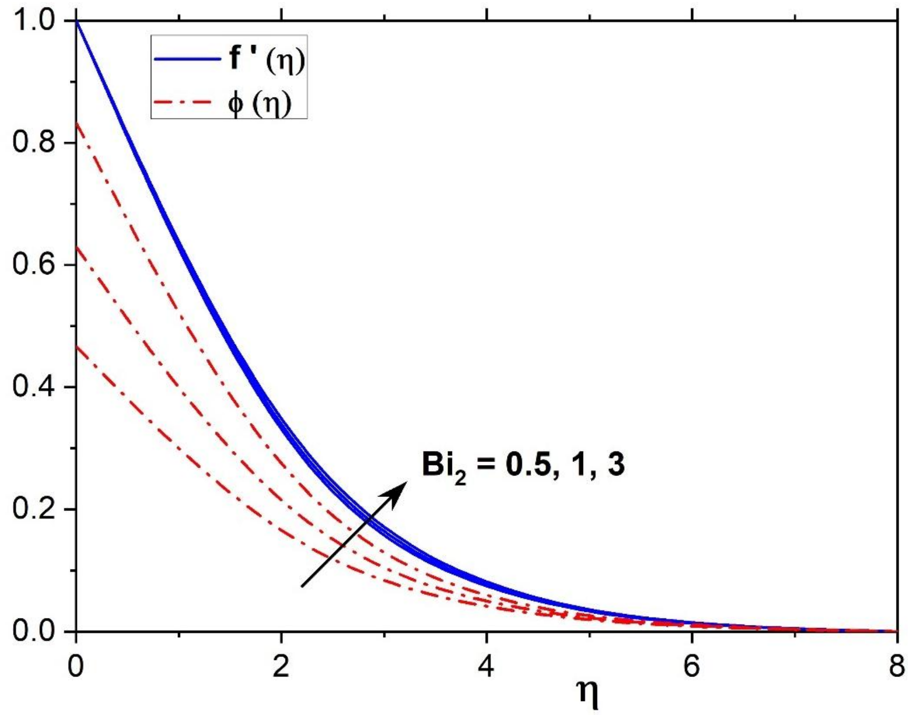

Figure 13: Influence of concentration Biot number on temperature profile.

The influence of the Prandtl number is presented in Figure 11, which reveals that fluids with larger Pr values exhibit noticeably lower temperature profiles. This trend arises because a higher Prandtl number corresponds to weaker thermal diffusivity, restricting heat penetration into the fluid and thereby thinning the thermal boundary layer. On the other hand, Figures 12 and 13 illustrate the impact of the thermal and solutal Biot numbers (Bi1, Bi2) on the velocity, temperature, and concentration distributions. It is evident that increasing either Biot number enhances the corresponding convective heat and mass exchange at the boundary surface. Consequently, all three boundary layer thicknesses momentum, thermal, and solutal expand with stronger Biot effects. This highlights the vital role of convective boundary conditions in controlling energy and species transport in practical applications.

https://doi.org/10.71058/jodac.v10i01008

Conclusion

The present analysis highlights the significant influence of various physical parameters on the velocity, temperature, and concentration profiles of Jeffrey liquid under magnetohydrodynamic and convective boundary conditions. It is found that the magnetic field suppresses fluid motion through Lorentz force effects, while higher Deborah numbers accelerate the flow due to diminished viscous resistance. Whereas buoyancy forces arising from mixed convection enhance the momentum boundary layer thickness. Heat and mass transfer characteristics are strongly affected by thermophysical effects such as Dufour and Soret numbers and Prandtl number. The results confirm that Dufour and Soret effects play a crucial role in coupling heat and mass fluxes, while larger thermal radiation intensify temperature fields through frictional heating. Thermal radiation, Biot numbers, and heat generation parameters further enlarge thermal and solutal boundary layers, demonstrating the importance of radiative and convective mechanisms in transport processes.

Table 1: Numerical results of $C_f Re_x^{1/2}, Nu_x Re_x^{-1/2}$ and $Sh_x Re_x^{-1/2}$ for different values of Bi_1, Bi_2, Df, Sr and Ec .

					$\boldsymbol{M = R = \beta = 0.4, \theta_w = 1.2, Q = \tau = 0.2, \lambda_1 = 0.8, N = S = Sc = 0.5, Pr = 0.71}, \lambda = 1\ \alpha = 0.9$		
Bi_1	Bi_2	Df	Sr	Ec	$C_f Re_x^{1/2}$	$\frac{Nu_x}{Re_x^{\frac{1}{2}}}$	$\frac{Sh_x}{Re_x^{\frac{1}{2}}}$
0.5	0.5	0.4	0.3	0.2	-1.31631	0.332205	0.267822
0.5					-1.31631	0.332205	0.267822
1					-1.05595	0.444246	0.268124
2					-0.83483	0.533657	0.268386
	0.5				-1.31631	0.332205	0.267822
	1				-1.20005	0.323166	0.372922
	2				-1.10043	0.315153	0.464938

https://doi.org/10.71058/jodac.v10i01008

		0.4			-1.31631	0.332205	0.267822
		0.8			-1.18913	0.300934	0.270648
		1.2			-1.06246	0.270075	0.273278
			0.3		-1.31631	0.332205	0.267822
			0.6		-1.30765	0.335312	0.259471
			0.9		-1.29859	0.33836	0.250998
				0.2	-1.31631	0.332205	0.267822
				0.4	-1.25161	0.312761	0.269197
				0.6	-1.19056	0.295087	0.270464

Table 2: Numerical results of $C_f Re_x^{1/2}, Nu_x Re_x^{-1/2}$ and $Sh_x Re_x^{-1/2}$ for different values of M, R, θ_w, Q, τ and λ_1.

						$Bi_1 = Bi_2 = 0.5, Df = \beta = 0.4, Sr = 0.3, Ec = 0.2, N = S = Sc = 0.5, \Pr = 0.71,\ \lambda = 1\ \alpha = 0.9$		
M	R	θ_w	Q	τ	λ_1	$C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$
0.4	0.4	1.2	0.2	0.2	0.8	-1.31631	0.332205	0.267822
0.2						-1.14975	0.340682	0.269274
0.4						-1.31631	0.332205	0.267822
0.6						-1.47204	0.323853	0.266475
	0.4					-1.31631	0.332205	0.267822
	0.8					-1.14239	0.41901	0.272073
	1.2					-1.00806	0.495378	0.275266
		1.2				-1.31631	0.332205	0.267822
		1.4				-1.24904	0.391893	0.269267
		1.6				-1.16858	0.464056	0.270959
			0.2			-1.31631	0.332205	0.267822
			0.4			-0.98168	0.249281	0.274909
			0.6			-0.45056	0.127033	0.284427
				0.2		-1.31631	0.332205	0.267822
				0.5		-1.32096	0.330943	0.272463

https://doi.org/10.71058/jodac.v10i01008

				0.8		-1.32539	0.329735	0.276922
					0.4	-0.93996	0.331735	0.269035
					0.8	-1.31631	0.332205	0.267822
					1.2	-1.72256	0.332447	0.266955

Table 3: Validation of present numerical results for $f''(0)$ in comparison with the benchmark studies of Hayat et al. [5] and Dalir [6] when $S = \lambda = 0, \beta = 0.2$

λ_1	Homotopy Analysis Method [5]	Integral kinetic boundary method [6]	Present study (RKF-45)
0	-0.91287	-0.91468	-0.91299
0.2	-1.00000	-1.00124	-1.00006
0.4	-1.08012	-1.08100	-1.08016
0.6	-1.15471	-1.15534	-1.15472
0.8	-1.22474	-1.22522	-1.22476
1.0	-1.29099	-11.2913	-1.29110
1.2	-1.35401	-1.35428	-1.35401
1.4	-1.41421	-1.41442	-1.41422
1.6	-1.47196	-1.47212	-1.47196
1.8	-1.52753	-1.52770	-1.52753
2.0	-1.58114	-1.58124	-1.58114

https://doi.org/10.71058/jodac.v10i01008

1. Turkyilmazoglu, M., and I. Pop. "Soret and heat source effects on the unsteady radiative MHD free convection flow from an impulsively started infinite vertical plate." *International Journal of Heat and Mass Transfer* 55, no. 25-26 (2012): 7635-7644.
2. Zheng, Lian-Cun, Xin Jin, Xin-Xin Zhang, and Jun-Hong Zhang. "Unsteady heat and mass transfer in MHD flow over an oscillatory stretching surface with Soret and Dufour effects." *Acta Mechanica Sinica* 29, no. 5 (2013): 667-675.
3. Reddy, P. Sudarsana, and Ali J. Chamkha. "Soret and Dufour effects on MHD convective flow of Al2O3–water and TiO2–water nanofluids past a stretching sheet in porous media with heat generation/absorption." *Advanced Powder Technology* 27, no. 4 (2016): 1207-1218.
4. Sampath Kumar, P. B., B. J. Gireesha, B. Mahanthesh, and R. S. R. Gorla. "Nonlinear Thermal Convection in Jeffrey Liquid Flow with Cross Diffusion Effects Past a Stretched Surface." *Diffusion Foundations* 11 (2017): 84-98.
5. Hayat, Tasawar, Ikram Ullah, Taseer Muhammad, and Ahmed Alsaedi. "Radiative three-dimensional flow with Soret and Dufour effects." *International Journal of Mechanical Sciences* 133 (2017): 829-837.
6. Zia, QM Zaigham, Ikram Ullah, Metal Waqas, A. Alsaedi, and T. Hayat. "Cross diffusion and exponential space dependent heat source impacts in radiated three-dimensional (3D) flow of Casson fluid by heated surface." *Results in physics* 8 (2018): 1275-1282.
7. Madan Kumar, R., R. Srinivasa Raju, M. Anil Kumar, and B. Venkateswarlu. "A numerical study of thermal and diffusion effects on MHD Jeffrey fluid flow over a porous stretching sheet with activation energy." *Numerical Heat Transfer, Part A: Applications* 86, no. 13 (2025): 4423-4444.
8. Ullah, Subhan, Ikram Ullah, Amir Ali, Kamal Shah, and Thabet Abdeljawad. "Investigation of cross-diffusion effect on radiative Jeffery-Hamel flow in convergent/divergent stretchable channel with Lorentz force and Joule heating." *Alexandria Engineering Journal* 86 (2024): 289-297.
9. Reddy, JV Ramana, K. Anantha Kumar, V. Sugunamma, and N. Sandeep. "Effect of cross diffusion on MHD non-Newtonian fluids flow past a stretching sheet with non-uniform heat source/sink: A comparative study." *Alexandria engineering journal* 57, no. 3 (2018): 1829-1838.

https://doi.org/10.71058/jodac.v10i01008

10. Revathi, Gadamsetty, Mamatha S. Upadhya, Raghunath Kodi, Dhananjay Yadav, Haribabu Kommaddi, and M. Jayachandra Babu. "Influence of cross-diffusion, couple stress, and non-Fourier heat flux on Jeffrey hybrid nanofluid flow and entropy generation in a vertical cylinder." *Phase Transitions* (2025): 1-22.
11. Noreen, Saima, Areeba Riaz, and Dianchen Lu. "Soret-Dufour effects in electroosmotic biorheological flow of Jeffrey fluid." *Heat Transfer* 49, no. 4 (2020): 2355-2374.
12. Raju, C. S. K., and N. Sandeep. "Heat and mass transfer in MHD non-Newtonian bio-convection flow over a rotating cone/plate with cross diffusion." *Journal of molecular liquids* 215 (2016): 115-126.
13. Seid, Eleni, Eshetu Haile, and Tadesse Walelign. "Multiple slip, Soret and Dufour effects in fluid flow near a vertical stretching sheet in the presence of magnetic nanoparticles." *International Journal of Thermofluids* 13 (2022): 100136.
14. Siddique, Imran, Muhammad Nadeem, Jan Awrejcewicz, and Witold Pawłowski. "Soret and Dufour effects on unsteady MHD second-grade nanofluid flow across an exponentially stretching surface." *Scientific Reports* 12, no. 1 (2022): 11811.
15. Ullah, Sharif, Ikram Ullah, and Amir Ali. "Soret and Dufour effects on dissipative Jeffrey nanofluid flow over a curved surface with nonlinear slip, activation energy and entropy generation." *Waves in Random and Complex Media* (2023): 1-23.
16. Reddy Gorla, Rama Subba, and Ibrahim Sidawi. "Free convection on a vertical stretching surface with suction and blowing." *Applied Scientific Research* 52, no. 3 (1994): 247-257.
17. T.Y. Wang, Mixed convection heat transfer from a vertical plate to non-Newtonian fluids" International Journal of Heat and Fluid Flow, 16 (1995) 56– 61.
18. A.J. Chamkha, Hydromagnetic three-dimensional free convection on a vertical stretching surface with heat generation or absorption, International Journal of Heat and Fluid Flow, 20 (1999) 84-92.
19. Rashidi, Mohammad Mehdi, Muhammad Ashraf, Behnam Rostami, Taher Mohammad Rastegari, and Sumra Bashir. "Mixed convection boundary-layer flow of a micro polar fluid towards a heated shrinking sheet by homotopy analysis method." *Thermal science* 20, no. 1 (2016): 21-34.
20. Gireesha, B. J., B. Mahanthesh, Rama Subba Reddy Gorla, and K. L. Krupalakshmi. "Mixed convection two-phase flow of Maxwell fluid under the influence of non-linear

https://doi.org/10.71058/jodac.v10i01008

thermal radiation, non-uniform heat source/sink and fluid-particle suspension." *Ain Shams Engineering Journal* 9, no. 4 (2018): 735-746.

21. F.M. Ali, R. Nazar, N.M. Arifin, I. Pop, Mixed convection stagnation-point flow on vertical stretching sheet with external magnetic field, Applied Mathematics and Mechanics, 35(2) (2014) 155-166.
22. M.Y. Malik, I. Khan, A. Hussain, T. Salahuddin, Mixed convection flow of MHD Eyring-Powell nanofluid over a stretching sheet: A numerical study, AIP Advances, 5 (2015) 117-118.
23. B. Mahanthesh, B.J. Gireesha and R.S.R. Gorla, Heat and mass transfer effects on the mixed convective flow of chemically reacting nanofluid past a moving/stationary vertical plate, Alexandria Engineering Journal, 55(1) (2016) 569–581.
24. S. Das, R.N. Jana, O.D. Makinde, Mixed convective magnetohydrodynamic flow in a vertical channel filled with nanofluids, Engineering Science and Technology, an International Journal, 18(2) (2015) 244-255.
25. K. Vajravelu, J.R. Cannon, J. Leto, R. Semmoum, N. Nathan, M. Draper, D. Hammock, Nonlinear convection at a porous flat plate with application to heat transfer from a dike, Journal of mathematical analysis and applications, 277 (2003) 609-623.
26. P.K. Kameswaran, P. Sibanda, M.K. Partha, P.V.S.N. Murthy, Thermophoretic and non-linear convection in non-Darcy porous medium, Journal of Heat Transfer, ASME, 136(4) (2014) 042601.
27. S. Shaw, G. Mahanta, P. Sibanda, Non-linear thermal convection in a Casson fluid flow over a horizontal plate with convective boundary condition, Alexandria Engineering Journal, 55(2) (2016) 1295–1304.
28. Jyoti, D. K., V. Nagaradhika, P. B. Sampath Kumar, and Ali J. Chamkha. "Nonlinear Convection and Radiative Heat Transfer in Kerosene-Alumina Nanofluid Flow Between Two Parallel Plates with Variable Viscosity." *Journal of Nanofluids* 13, no. 5 (2024): 1055-1062.
29. Shoaib, Muhammad, Shafaq Naz, Muhammad Asif Zahoor Raja, Khadeeja Arshad, Iftikhar Ahmad, and Kottakkaran Sooppy Nisar. "Numerical treatment for nonlinear mixed convection and thermal radiative Newtonian fluid flow system." *Journal of Radiation Research and Applied Sciences* 18, no. 3 (2025): 101675.
30. T. Hayat, Z. Iqbal, M. Mustafa, A. Alsaedi, Unsteady flow and heat transfer of Jeffrey fluid over a stretching sheet, Thermal Science, 18(4) (2014) 1069-1078.

https://doi.org/10.71058/jodac.v10i01008

31. N. Dalir, Numerical study of entropy generation for forced convection flow and heat transfer of a Jeffrey fluid over a stretching sheet, Alexandria Engineering Journal, (2014), http://dx.doi.org/10.1016/j.aej.2014.08.005.
32. M. Qasim, Heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source/sink, Alexandria Engineering Journal, 52(4) (2013) 571-575.
33. S.A. Shehzad, F.E. Alsaadi, S.J. Monaquel, T. Hayat, Soret and Dufour effects on the stagnation point flow of Jeffrey fluid with convective boundary condition, The European Physical Journal Plus, 128 (2013) 56.
34. Ramesh, G. K. "Numerical study of the influence of heat source on stagnation point flow towards a stretching surface of a Jeffrey nanoliquid." *Journal of Engineering* 2015, no. 1 (2015): 382061.
35. Krishna, M. Veera, and Ali J. Chamkha. "Hall and ion slip effects on magnetohydrodynamic convective rotating flow of Jeffreys fluid over an impulsively moving vertical plate embedded in a saturated porous medium with Ramped wall temperature." Numerical Methods for Partial Differential Equations 37, no. 3 (2021): 2150-2177.