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SYSTEM IDENTIFICATION OF UAV DYNAMICS USING ECHO STATE NETWORKS AND CLASSICAL METHODS

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Abstract: *System identification plays a crucial role in determining the behavior and dynamics of flying objects and in modeling their dynamic systems. The availability of a suitable model of UAV can reduce some of the challenges of a vehicle control system designing. The selection of the appropriate method for identifying a system depends on the system structure and its complexity. Therefore, according to these conditions, in this paper, classical methods of system identification and Echo State Networks (ESNs) are employed to identify the dynamics of a small fixed-wing UAV. To evaluate the accuracy of the identification methods used, system output comparisons and transfer function coefficient analyses are conducted using appropriate metrics. The results indicate that the Least Squares method provides the best unbiased estimator under noise-free conditions, while the ESN and some of the classical methods examined demonstrate robust performance in noisy environments.*

Keywords: System identification, UAV, least square, Echo state networks, Root mean square.

1. Introduction

The use of UAVs in various applications such as architecture engineering, construction, modern agriculture, smart cities and Humanitarian Relief has expanded greatly in recent years [1 - 4]. System Identification is the process of constructing a mathematical model of a dynamic system based on input and output data. In UAV system identification, the dynamic model of the system is extracted by applying excitation signals to the system inputs and measuring the outputs [5, 6]. This process is crucial for controller design, system behavior analysis, and simulation [9]. In the identification process, the model structure and the selection of the appropriate model are of great importance and can have a significant impact on the appropriate description of the system [7].

Various techniques have been employed for identifying linear and nonlinear systems. Classical methods such as least squares and structured models like ARX/ARMAX, as well as neural-network-based approaches, all exhibit limitations. For example sensitivity to model-structure selection, large data requirements, and, in some cases, limited ability to capture complex nonlinear behavior. Traditional identification techniques are often formulated in an asymptotic framework, whereas other approaches, such as maximum likelihood estimators, are typically considered in a non-asymptotic or finite-sample setting. Consequently, choosing an appropriate identification method depends on multiple factors, including implementation simplicity and the complexity of the underlying system dynamics [9].

Various methods have been developed to identify nonlinear and linear dynamics of UAVs. In this paper, the linear longitudinal dynamics of a small fixed-wing UAV are identified using several least squares based approaches and an echo state network (ESN), both operating in the time domain. The aforementioned identification methods are compared in various aspects, especially under noisy and noise-free conditions.

The structure of this paper is as follows: In Section 2, the dynamics of the aircraft are described and identification methods are defined. In Section 3 the identification procedure and simulation scenarios has been explained. In Section 4, the results obtained from UAV identification are presented and the results are compared using two indices RMS and NPE Methods and discussed. In Section 5, the conclusion of paper are presented.

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2. Theory

This section is about introducing the dynamics of the UAV used in the research and categorizing the system identification methods. In the following, the basics of the LS identification method, other LS methods, and the ASN method are explained.

2.1. Aircraft Longitudinal Model

The aircraft studied in this paper is a small fixed-wing UAV that has been used in reference [8] and is shown in Figure 1. The mathematical model of this UAV is linearized around the trim conditions and in this study, the linear longitudinal state space equations of the aircraft, which are in the form of equation (1), have been used. The longitudinal dynamics of the aircraft have control operators of the elevator with a maximum deviation angle of 15 degrees and throttle (as a percentage of the throttle deviation from the trim conditions in percent). Also, in the given state space equations, 5 state variables [u, w, q, θ, h] have been selected.

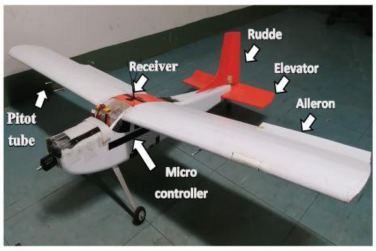

Fig. 1. Small fixed wing UAV [8]

$$\dot{x}_{long} = A_{long}x_{long} + B_{long}u_{long} \tag{1}$$

$$A_{long} = \begin{bmatrix} X_u & X_w & X_q & -g & 0 \\ Z_u & Z_w & Z_q & 0 & 0 \\ M_u & M_w & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ sin\theta^* & -cos\theta^* & 0 & u_0 cos\theta^* + w_0 sin\theta^* & 0 \end{bmatrix}$$
$$B_{long} = \begin{bmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & Z_{\delta_t} \\ M_{\delta_e} & M_{\delta_t} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

Assuming a small pitch angle, the longitudinal dynamics of the vehicle is converted to the following form and used:

$$A_{long} = \begin{bmatrix} X_u & X_w & X_q & -g & 0 \\ Z_u & Z_w & Z_q & 0 & 0 \\ M_u & M_w & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}, B_{long} = \begin{bmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & Z_{\delta_t} \\ M_{\delta_e} & M_{\delta_t} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

The numerical values of the derivatives of aircraft stability and control are substituted and the longitudinal dynamics of the UAV are extracted as follows:

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$$A_{long}=\begin{bmatrix}-0.1605 & 51.263 & 3.3652 & -9.81 & 0\\ -0.1604 & 0.5654 & 0.2739 & 0 & 0\\ 2.4652 & -8.678 & -1.281 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & -1 & 0 & 0.2985 & 0\end{bmatrix}, B_{long}=\begin{bmatrix}0.3684 & 0.1097\\ 0.3914 & 0.1431\\ -6.037 & -2.129\\ 0 & 0\\ 0 & 0\end{bmatrix} \quad (4)$$

The equations of longitudinal motion of the aircraft in continuous space are entered into the form of equation (4) and then tidied into the form of discrete state space with a sampling time of 0.01 seconds. The system transfer function is extracted in a decoupled form (the ratio of each output to each control input). The system has two control inputs and five state variables; therefore, ten discrete transfer functions are extracted. It is observed that some of the transfer functions have a degree 4 denominator and a degree 3 face, and some others have a degree 5 denominator and a degree 4 face, which indicates the number of past inputs and outputs (memory) used in calculating the future output.

In general, in this system, it is assumed that all state variables are measured and are the output of the system. Therefore, in a real system, some variables cannot be measured and need to be estimated using state observers.

2.2. Classification of Identification Methods

Classical system identification is generally divided into two categories: data-driven (or nonparametric) methods, and prior knowledge-based (or parametric) methods. The terms "data-driven" and "nonparametric" indicate that only limited prior knowledge is used—typically assumptions about the system's linearity, nonlinearity, or time dependence. In contrast, parametric identification incorporates more detailed prior knowledge about the system and its dynamic relationships during the identification process [14]. Also, System identification methods are classified into two domains: time-domain and frequency-domain. Although both domains contain equivalent information about the system, accessing specific features may be easier in one domain than in the other. Consequently, the choice of identification method depends on which domain offers more practical advantages for extracting the desired system characteristics.

Common examples include LS and recursive algorithms for parametric time-domain identification, impulse response and correlation analysis for nonparametric time-domain approaches, and spectral or frequency response analysis for nonparametric frequency-domain estimation.

In this study, we focus on least-squares methods and echo state networks, which are time domain parametric methods and described in detail in the following subsections.

2.3. Linear Regression and Least Square Method

The content in this section is adapted from reference [10]. Linear regression provides a basic structure for dynamic system modeling. In this structure, the output is generated using known signals and unknown parameters. In its standard form, the model can be written as:

$$y(t)=\emptyset^T(t)\theta+e(t) \quad (5)$$

where $y(t)$ is the system output, $\emptyset(t)$ is a regressor vector constructed from past input and output data, θ is the unknown parameter vector, and $e(t)$ represents the modeling error or noise.

Collecting data points yields the compact matrix form:

$$y=\Phi\theta+e \quad (6)$$

Where y is an output vector, Φ is the regression matrix, and e is the error vector.

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To estimate the parameter vector, the least squares method minimizes the quadratic cost function:

$$J(\theta) = \frac{1}{2}\|y - \Phi\theta\|^2 \tag{7}$$

Taking the derivative of $J(\theta)$ with respect to θ and setting it to zero leads to the normal equations:

$$\Phi^T\Phi\theta = \Phi^T y \tag{8}$$

The optimal parameter estimate is then obtained as:

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T y \tag{9}$$

The least squares approach is widely used in system identification due to its simple structure, efficient computation, and good numerical stability. It forms the basis for many classical model structures such as ARX and ARMAX models, and serves as a starting point for more advanced estimation and regularization techniques.

2.4. Other Methods of LS

The Least Squares family of methods includes parametric fitting algorithms tailored to different practical needs. Recursive Least Squares is a recursive version that updates parameters online and sometimes uses a forgetting factor to track system changes over time, making it suitable for identifying time-varying systems and applications requiring fast response [12].

The Recursive Extended Least Squares (RELS) algorithm is a recursive method that estimates parameters from current and past input-output data and updates the estimated residuals in the process. This algorithm can build an over-parameterized model in a nonlinear system and enables efficient computation. At each time step, RELS minimizes a quadratic cost function and applies matrix inversion techniques to update the gain and covariance matrices. This method guarantees convergence under continuous excitation and exhibits good performance for real-time identification of nonlinear and linear systems with colored noise [13].

2.5. Echo State Networks

ESN networks with three input, hidden, and output layers are a type of recurrent neural networks that have the property of fast and accurate learning by randomly and consistently weighting the internal connections of the hidden layer and weighting the output layer using linear regression. This method that is using in identification procedure in article is described in reference [11].

This structure that shows in Figure 2, makes the training process very fast, since only one layer needs to be trained. To ensure network stability, it is essential to carefully tune key hyper-parameters such as the spectral radius, input scaling factor, and reservoir sparsity, as these directly influence the dynamic behavior and memory capacity of the system. The activation function in the reservoir layer is usually chosen to be nonlinear (such as hyperbolic tangent or sigmoid) in order to model more complex features of the input.

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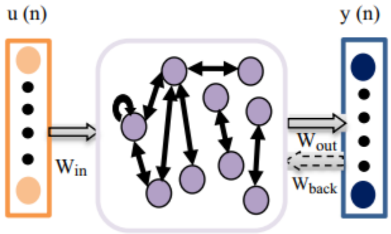

Figure 2. ESN Structure [11]

The formulation of updating the state and output are as follows:

$$x(n+1) = f(Wx(n) + W_{in}u(n+1) + W_{back}y(n))$$
$$y(n+1) = f_{out}(W_{out}[x(t+1), u(t+1), y(t)] + W_{out}^{bias}) \tag{10}$$

The connection weights From the input layer to the reservoir layer are W_{in}, where W is the connection weights of the reservoir neurons. The connection weights from hidden layer to output layer is W_{out} (bias weights are denoted by W_{out}^{bias}). An optional feedback connection from the output layer to the reservoir, denoted by W_{back} (optional feedback in Figure 2).

3. Methodology Procedure

The order of the numerator and denominator of the transformation functions for identification is considered to be in accordance with the order of the real system. This is intended for accurate comparison of the transformation function parameters. To identify the UAV system introduced in the study, the definition of 1000 random PRBS inputs and recording the corresponding output have been used to define the required data set. This issue has been considered in various identification methods, in which data generation in the range of changes from 10 to 80 percent and in the range of negative and positive 10 for the elevator deviation in terms of angle and throttle in terms of percentage has been used. Also, the instrument noise applied to the system has been considered as zero in noise free condition and 5 percent for noisy condition.

In this paper, time-domain parametric methods have been employed for system identification. The first approach is the least-squares (LS) method using an ARX (Autoregressive with Exogenous input) model. Subsequently, recursive LS (RLS), recursive extended LS (RELS), and least mean squares (LMS) algorithms have been applied to identify the system. Each method has been simulated under both noise-free and noisy conditions, with variations in certain model features. Finally, the echo state network (ESN) method has been used to perform system identification in the last part of the study.

The objectives of simulating and comparing the above methods for UAV system identification are as follows:

- Discretize the system's dynamic model for identification in the discrete-time domain.
- Use random input signals to excite all modes of the system, thereby minimizing the risk of ill-conditioning in the regressor matrix and improving the accuracy of identification.
- Compare the performance of LS, RLS, LMS, and RELS algorithms in estimating the dynamic parameters and producing outputs that closely match the actual system response.
- Compare the algorithms' performance under both noise-free and noisy conditions to evaluate robustness and accuracy.

4. Results and Discussion

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4.1. Results obtained by LS

In this section, system identification is performed using the Least Squares (LS) method under both noiseless and noisy conditions. A random input signal was applied to the system to excite all modes, and the resulting output was used along with the input data to construct the regressor matrix. The LS algorithm was then employed to estimate the system parameters. In Figures 3 and 4, a comparative plot illustrates the performance of LS in identifying the system across the two scenarios by comparing the output of true and identified system. In table. 1, the estimated transfer function parameters of the system corresponding to the elevator input and the longitudinal velocity output u, were extracted and compared with those of the true system in noise-free condition. Additionally, the identification error, measured by the Mean Squared Error (MSE), is presented in this table. In the noiseless case, the MSE approaches zero, indicating highly accurate parameter estimation. In this case, the LS method successfully identified all model parameters with high precision, and the simulated output closely matched the actual system response. Note that, According to large number of system parameters, the MSE criterion in this method is obtained as an example and is not used in other methods in this article. Therefore, the comparative chart in Figure 17 is used to compare the methods.

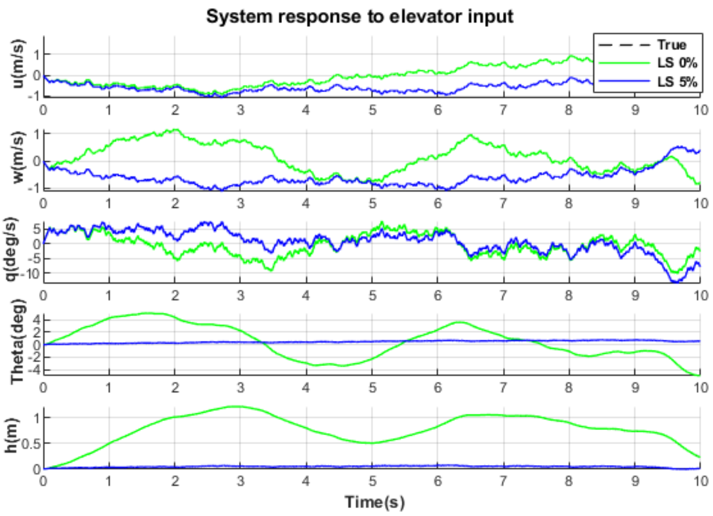

Figure 3. System output relative to elevator in LS

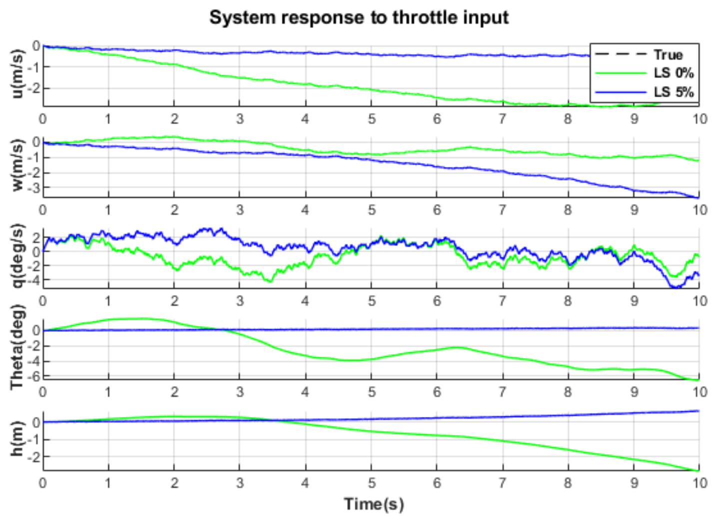

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Figure 4. System output relative to Throttle in LS

Table. 1. Comparing transfer function parameters in system and LS results for elevator input and $u(m/s)$ output

	True	Identified	MSE
Numerator	0 0.0036 -0.011 0.011 -0.0036	0 0.0036 -0.011 0.011 -0.0036	$1.04767e^{-12}$
Denominator	1 -3.9911 5.9735 -3.9736 0.9912	1 -3.9911 5.9735 -3.9736 0.9912	$1.04767e^{-12}$

To reduce the noise effect in identification, a low-pass filter (such as a moving average low-pass filter) is added to the LS in noisy condition in previous stage. The results illustrate that the Adding the filter had not Significant impact on the convergence of the identified model and true system outputs.

4.2. Results obtained by RLS

The effect of comparing the performance of RLS in system identification of noise-free and noisy system has been investigated in this section. The comparative output diagram is shown in Figs. 5 - 6. In terms of comparison of the identified output with the true output, it is observed that the RLS algorithm has good performance in noise-free conditions. Of course, the LS method, which was investigated in Figs. 3 and 4, has a complete correspondence with the real output in terms of the parameters of the transfer function and similarity in noise-free conditions, and the RLS method has a small error. Also, the presence of measurement noise has a small effect on identification with this method.

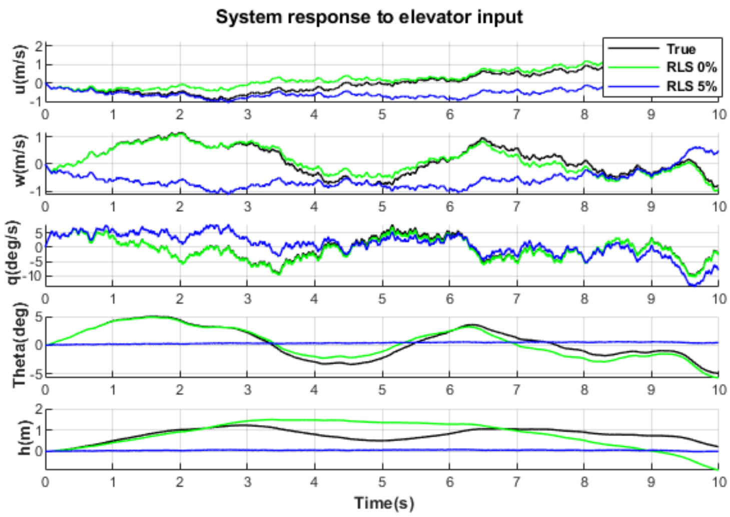

Figure 5. System output relative to elevator in RLS

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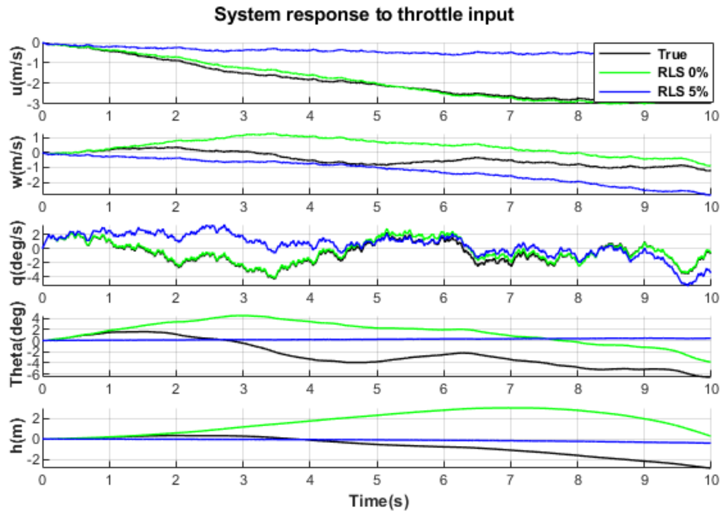

Figure 6. System output relative to Throttle RLS

In this section, the effect of applying the forgetting factor on system identification has been investigated. The results are presented in Figs. 7 and 8. The effect of the forgetting factor has been investigated only for the noisy case to investigate the effect of this factor in improving identification in noisy conditions. It is observed that the forgetting factor used has a small effect on reducing the effect of noise on matching the identified output with the real output and has acted almost similarly to the standard RLS and has even been associated with a decrease in accuracy in vertical speed relative to both inputs and in the pitch angle relative to the elevator. Therefore, the stability of this method is not guaranteed. Of course, one solution is to use another algorithm to update P, for example, it is predicted that updating with the Joseph algorithm can increase the capability of this factor. The result obtained from this section showed that in this case the forgetting factor does not have a suitable effect on identification.

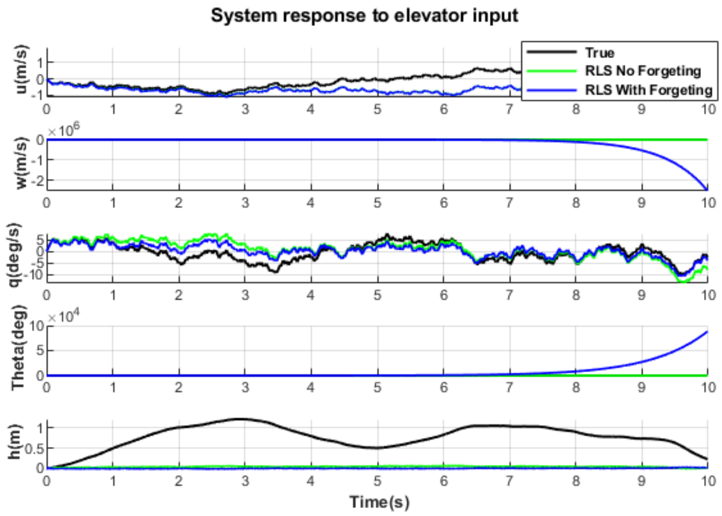

Figure 7. System output relative to elevator in noisy RLS with forgetting factor

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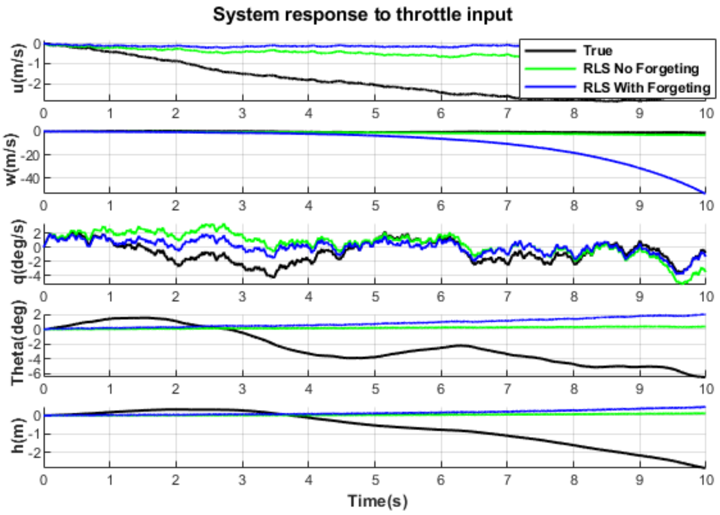

Figure 8. System output relative to throttle in noisy RLS with forgetting factor

4.3. Results obtained by LMS

The identification method using LMS is simulated in this section. The results of the identified output compared to the real system are presented in Figs. 9 - 10. In general, this algorithm has been able to identify the actual output with relatively good accuracy. The learning rate value γ for this algorithm has been set to 0.0018 using trial and error, which has enabled it to provide similar values to the noise-free state in terms of RMS and NPE criteria in Figure 17, under conditions of measurement noise. Therefore, it can be said that it has been able to neutralize the effect of noise in identification.

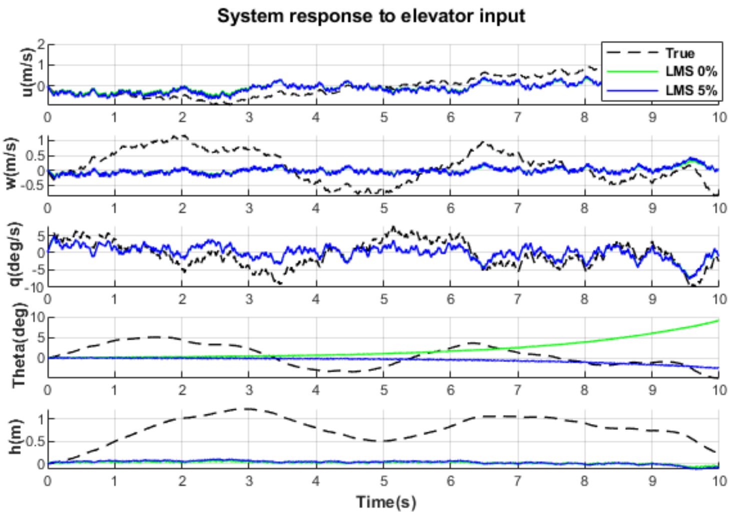

Figure 9. System output relative to elevator in LMS

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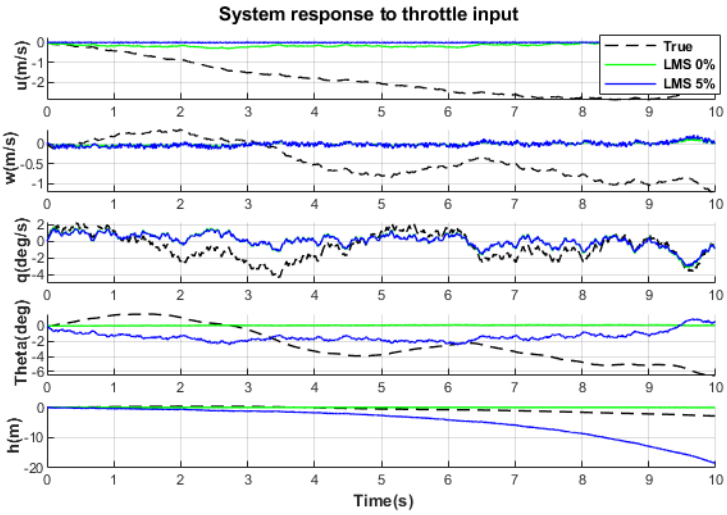

Figure 10. System output relative to throttle in LMS

4.4. Results obtained by RELS

In this section, the identification of the UAV system using the RELS method has been designed and investigated. A comparative display of the output of the identified system and the real system is shown in Figs. 11 and 12. In general, the RELS method has been able to provide acceptable identification of the real output, and the advantage of this method has been its low sensitivity to noise, which has shown that in approximately 8 plotted state variables, this method has good resistance to measurement noise

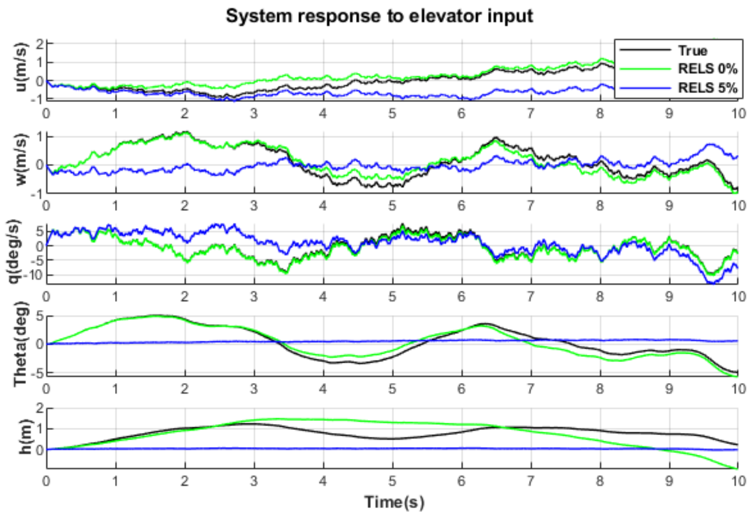

Figure 11. System output relative to elevator in RELS

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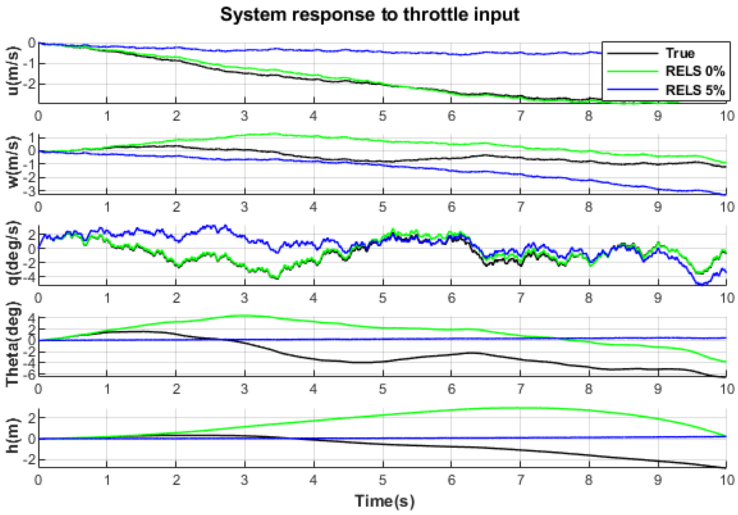

Figure 12. System output relative to throttle in RELS

Considering that the RELS is a suitable method for identifying colored noise, in this section the effect of this method in identifying colored noise has been examined. The method for generating colored noise is similar to the previous section, with the difference that this noise is not random in nature and is defined with specific characteristics. The results of comparing the output of the real system and the identified system in noise-free and colored noise conditions are shown in Figs. 13 and 14. In these graphs, the almost appropriate identification of the system in conditions with colored noise can be seen.

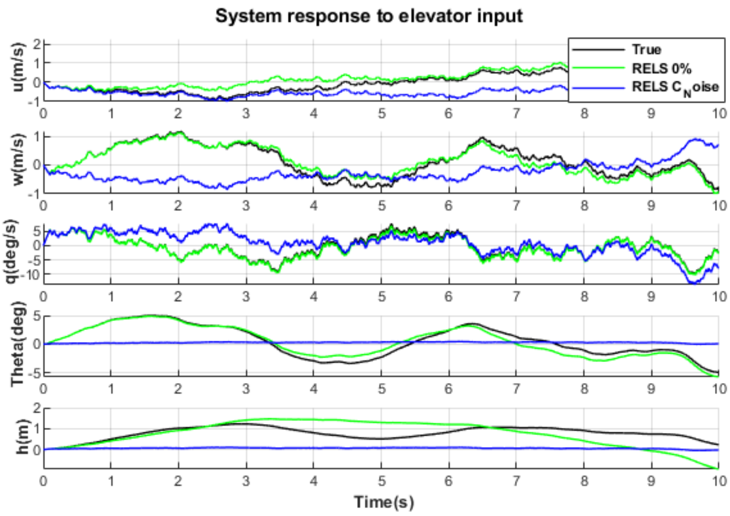

Figure 13. System output relative to elevator in RELS with colored noise

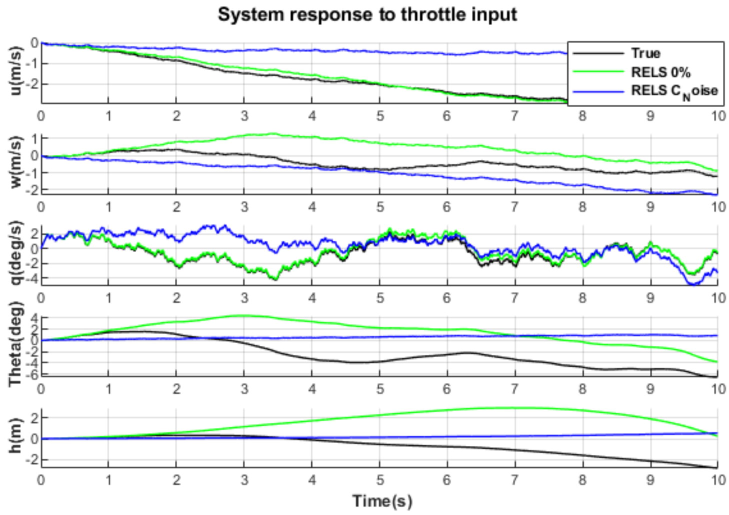

Figure 14. System output relative to throttle in RELS with colored noise

4.5. Results obtained by ESN

We used an Echo State Network (ESN) to perform linear identification of the system by training only the output weights with linear regression. This method usually works well in noisy conditions. The identification output diagrams with this method is shown in Figs. 15 - 16. Comparing the output diagrams of the system in the real system and the system identified with this neural network method shows that this method, despite being linear, has performed relatively good identification. Also, the effect of noise in identification with this method is very small, which indicates the low noise effect in identification with this method.

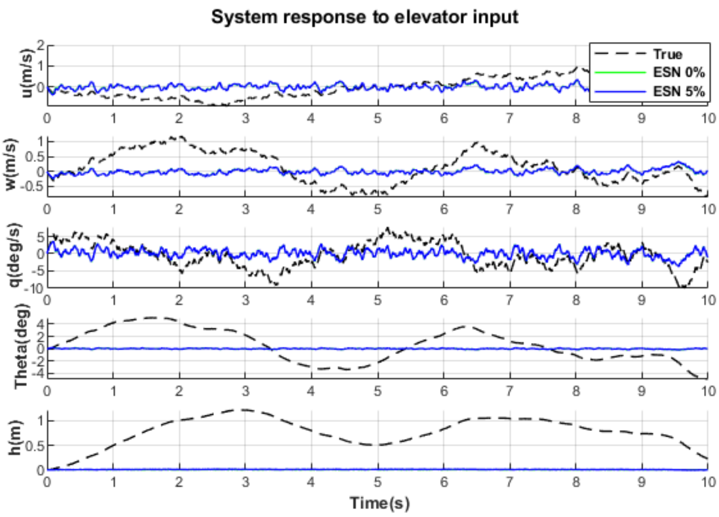

Figure 15. System output relative to elevator in ESN

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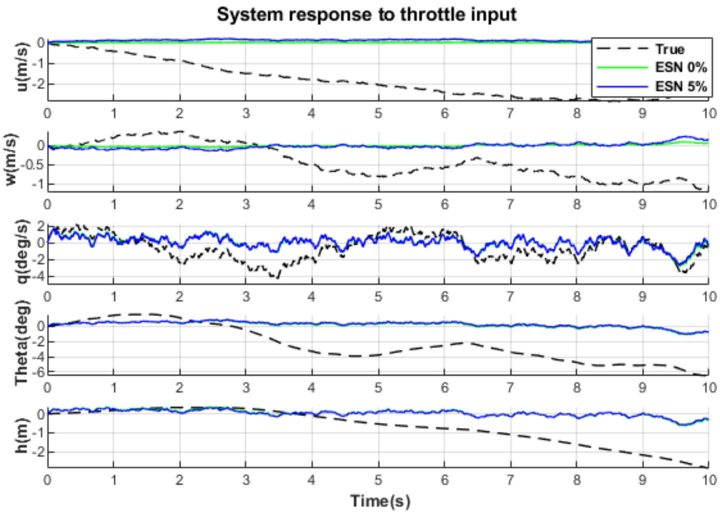

Figure 16. System output relative to throttle in ESN

4.6. Comparison of methods

In this section, the performance of the identification methods used in the research is compared with each other using the following two criteria:

- Comparison of the degree of correspondence of the identified output with the actual output using the RMSE (Root Mean Square Error) criterion for all parameters of dynamic model, which is abbreviated as RMS here.
- Comparison of the degree of correspondence of the parameters of the transformation functions using the Normalized Parameter Error (NPE) criterion

By comparing the values obtained for the comparison criteria in each identification method in the chart in figure 17, useful results can be extracted from this section

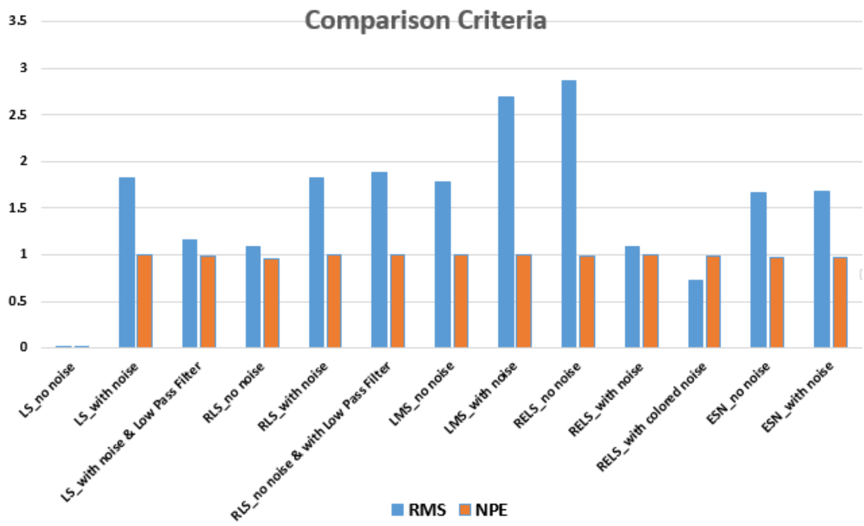

Figure (17): Comparison chart of identification methods

5. Conclusion

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In this paper, the effectiveness of various least squares-based algorithms in identifying the longitudinal dynamics of a small fixed-wing UAV was investigated. Based on the results presented in Figs. 3 -14 and figure 17, the LS method achieved a near-perfect match in noise-free conditions, offering the most accurate identification. However, its performance deteriorates significantly in the presence of measurement noise. The RLS method provided reasonably accurate parameter and output identification and demonstrated greater robustness to noise compared to LS. The forgetting factor, in this specific case, did not yield notable improvements under noisy conditions.

Beyond their strong performance in noise-free scenarios, both RELS and LMS methods also delivered satisfactory results in noisy environments, making them the most effective among the tested algorithms under such conditions. The LMS method, in particular, showed good adaptability in handling colored noise. Overall, each LS-based method exhibits distinct behavior depending on its structural characteristics and the noise conditions.

While the ESN method providing moderate accuracy in noise-free identification, maintains its performance in noisy conditions and is less affected. According to the comparative bar chart, The LS method has performed excellently in noise-free conditions and Compared to other methods, the RELS method performs better in identifying the system when the data is noisy.

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