A NATURAL FREQUENCY-BASED TECHNIQUE TO DETECT STRUCTURAL CHANGES

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Abstract: *Structural changes within materials often indicate potential damage caused by design flaws, fabrication errors, overloading, natural events, or aging. Cracks, a common type of defect resulting from fatigue or mechanical stress, can propagate under cyclic loading and lead to structural failure if undetected. Monitoring natural frequency variations provides a reliable, non-destructive method for detecting such defects. This study investigates the influence of crack size on the natural frequencies of a cantilever beam using analytical, numerical, and experimental approaches. Analytical modeling is based on Euler–Bernoulli beam theory, numerical simulations are carried out in COMSOL Multiphysics, and experimental validation is performed using a copper cantilever beam subjected to both free and forced vibration tests. Results show that increasing crack size reduces the natural frequency due to local stiffness degradation. The close agreement among analytical, simulation, and experimental findings demonstrates the effectiveness of frequency-based monitoring as a practical tool for structural health assessment.*

Keywords: *Vibration, Condition Monitoring, Structural Health, Crack Detection, Natural Frequency, Cantilever Beam*

1. Introduction

Structural health monitoring (SHM) plays a crucial role in ensuring the safety and reliability of engineering systems. Among various SHM techniques, vibration-based methods are widely adopted due to their ability to identify stiffness variations caused by damage such as cracks.

When a structure develops cracks, its dynamic characteristics—stiffness, damping, and natural frequencies—change significantly. A reduction in stiffness results in a measurable decrease in natural frequency, which can be effectively used as a damage indicator.

The crack behavior can be modeled using fracture mechanics, where the local stiffness reduction is represented by a flexural spring derived from Castigliano's second theorem. This enables analytical and numerical prediction of vibration response for damage identification.

Previous researchers have contributed extensively to vibration-based crack detection. Orhan *et al.* [1] analyzed free and forced vibration of cracked cantilever beams to identify crack depth and location. Chasalevris and Papadopoulos [2] proposed a local compliance matrix for beams with multiple transverse cracks. Nahvi and Jabbari [3] developed analytical and experimental methods for crack identification using normalized frequency contours. Yang et al. [4] presented an energy-based model to evaluate stiffness changes due to open cracks. Dharmaraju et al. [5] utilized finite element models to predict the influence of cracks on beam vibration. Ruotolo et al. [6] examined forced vibration behavior of cantilever beams with open and closed cracks. Loutridis et al. [7] applied empirical mode decomposition for breathing crack detection. Ertugrul et al. [8] combined experimental and numerical vibration analyses for crack localization. Building on

these studies, this work aims to analyze the vibration response of a copper cantilever beam with circular cracks of varying sizes through analytical, simulation, and experimental approaches.

2. Analytical investigation of natural frequencies of the cantilever beam

According to the Euler-Bernoulli beam theory, the bending moment in a beam is proportional to the curvature of the beam, and the curvature is proportional to the second derivative of the deflection of the beam with respect to the beam's length.
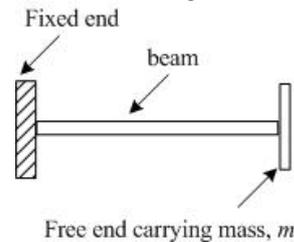

Figure 1. A cantilever beam with tip mass

A cantilever beam with rectangular cross-section is shown in Figure 1. Bending vibration can be generated by giving an initial displacement at the free end of the beam. The fundamental undamped circular natural frequency of the system is given as [9],

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} \qquad (1)$$

Where, m_{eq} is an equivalent mass placed at the free end of the cantilever beam and k_{eq} is the equivalent stiffness. The undamped natural frequency is related with the circular natural frequency as,

$$f_n = \frac{\omega_n}{2\pi} \qquad (2)$$

If any contacting type of transducer is used for the vibration measurement, it should be placed at end of the beam and then the mass of transducer must be added into the equivalent mass of the beam at the free end of the beam during the natural frequency calculation. If m_t is the mass of transducer, then the total mass at the free end of the cantilever beam is given as [10],

$$m_{eq} = \frac{33}{140}m_b + m_t \qquad (3)$$

Where m_b is the mass of beam and is given as,

$$m_b = \rho v = \rho b d l \qquad (4)$$

Where, ρ is the mass density of the beam material and for copper it is 8960 kg/m^3 and v is the volume of the beam from the fixed end to the free end ($l \times b \times h = 0.09 \times 0.0254 \times 0.0005$ m). The calculated natural frequency obtained as 37.9 Hz.

3. Simulation of Cantilever Beam

The free vibration test is performed in COMSOL by selecting Eigen-frequency field. The material and geometric properties of the beam are taken same as in analytical study.
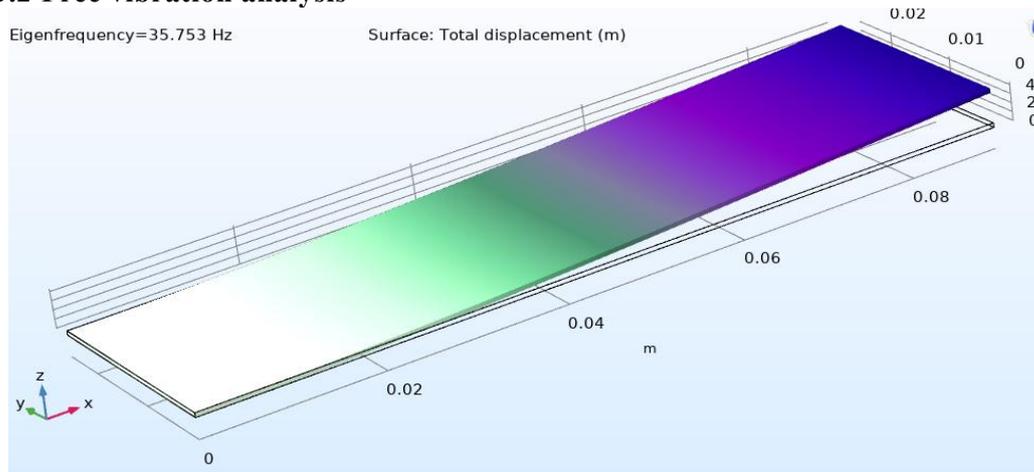
3.2 Free vibration analysis

Figure 2. Frequency response of free vibration analysis without crack

Figure 2 shows the first mode shape of the cantilever beam. The first mode frequency is found to be 35.753 Hz.

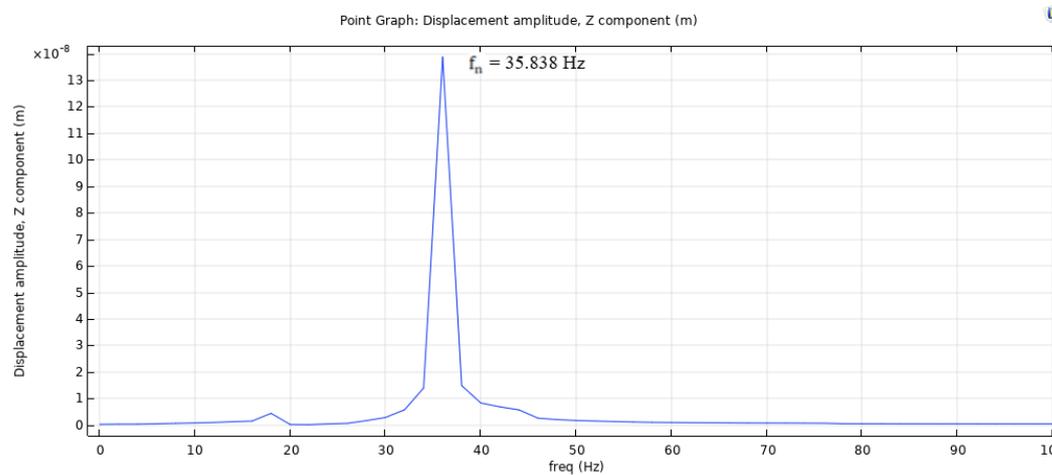## 3.2 Forced vibration analysis

Figure 3. Frequency response of forced vibration analysis without crack

The same beam is now tested for forced vibration with input harmonic base excitation acceleration of 0.5 g. Figure 3 shows the frequency response plot and the resonance frequency is found to be at 35.838 Hz. Further, the following section discusses the estimating the natural frequency of cantilever beam with circular shaped different sized cracks.

The following section discusses the studies on finding the natural frequency of cantilever beam with crack of different sized circular shapes.

4. Simulation of Cantilever Beam with Circular Cracks

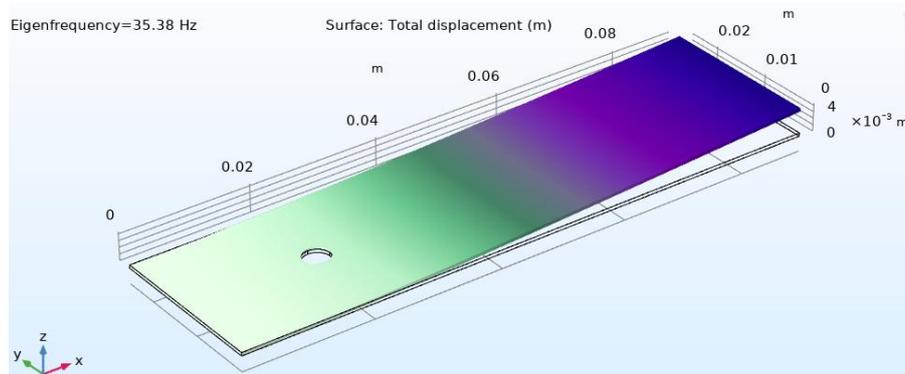### 6.1 Free vibration analysis

Figure 4. Frequency response of free vibration analysis with crack (4 mm circular crack)
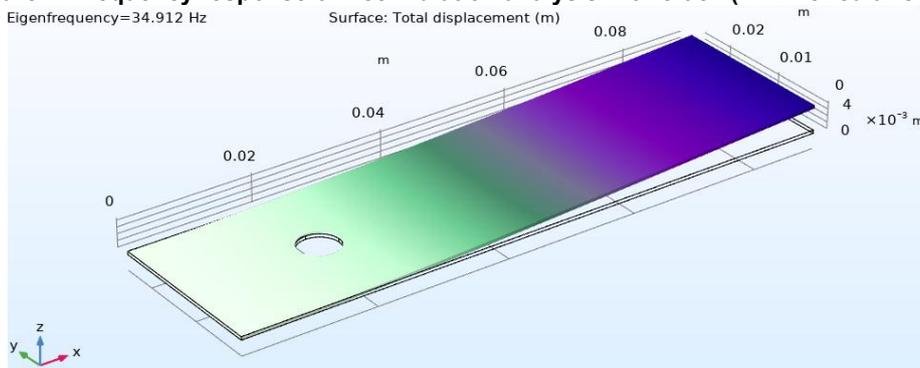

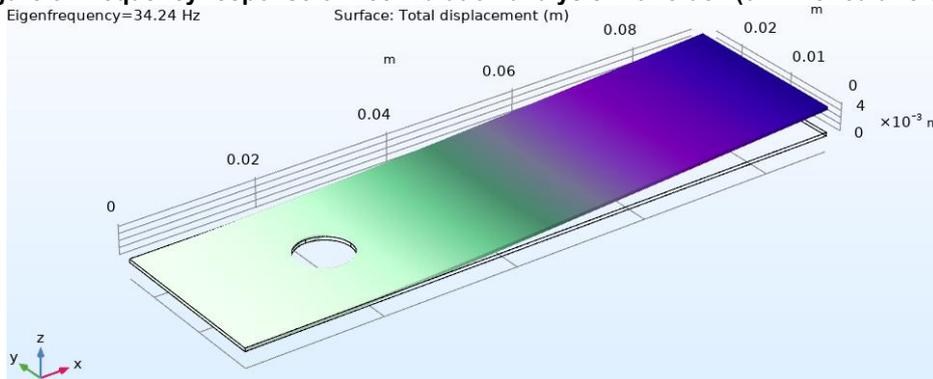**Figure 5. Frequency response of free vibration analysis with crack (6 mm circular crack)**

Figure 6. Frequency response of free vibration analysis with crack (8 mm circular crack)

The eigen frequency analysis is performed for three different sized holes viz. 4, 6 and 8 mm located at 20 mm from the fixed end of the cantilever beam. Figures 4, 5 and 6 shows the pictures of eigen frequency response. It is found from the simulations that for 4mm circular crack the natural frequency of beam is found to be 35.38 Hz and 34.912 Hz and 34.24 Hz for 6mm and 8mm circular cracks.
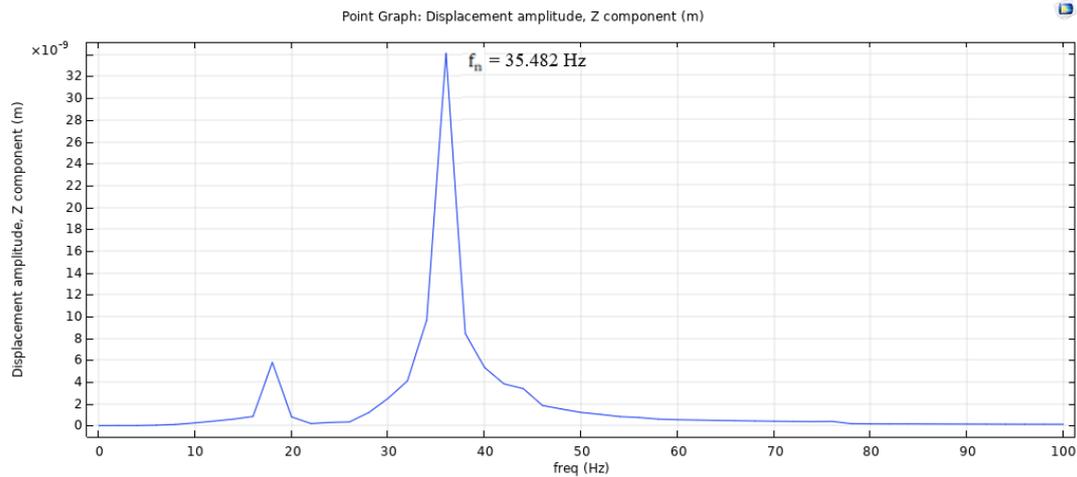
6.1 Forced Vibration Analysis

Point Graph: Displacement amplitude, Z component (m)

Figure 7. Frequency response of forced vibration with crack (4 mm circular crack)

Point Graph: Displacement amplitude, Z component (m)
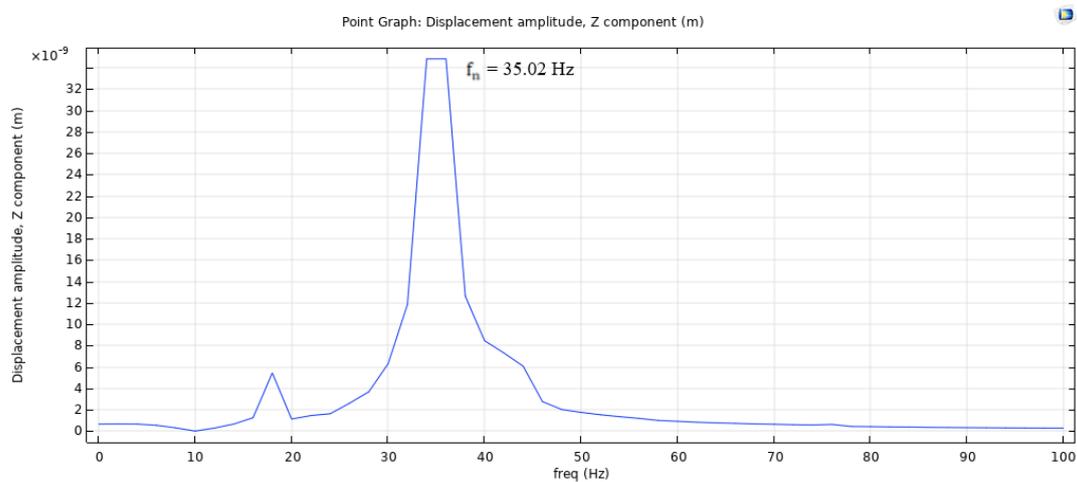
Figure 8. Frequency response of forced vibration with crack (6 mm circular crack)

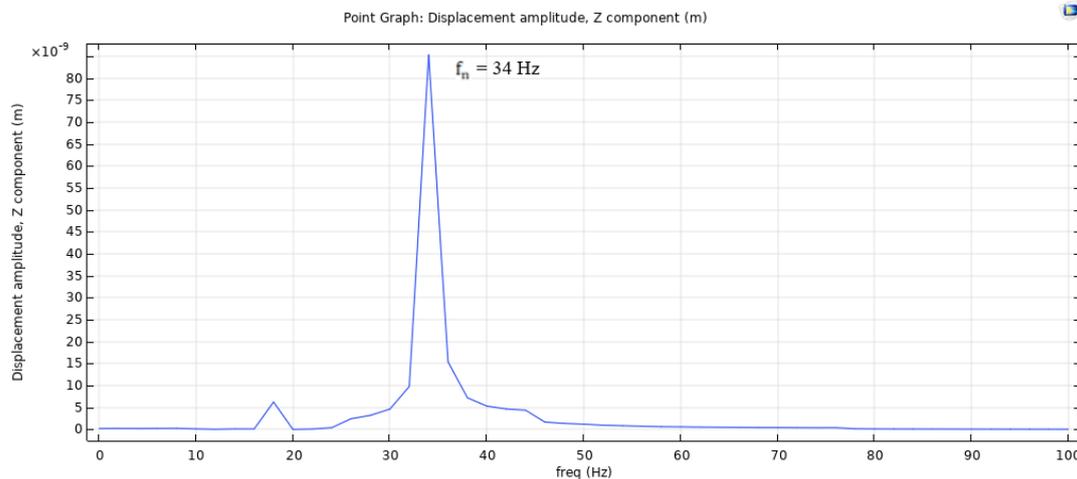Point Graph: Displacement amplitude, Z component (m)

Figure 9. Frequency response of forced vibration with crack (8 mm circular crack)

Now the circular shaped cracked beam is tested for its frequency response with base excitation of 0.5 g by sweeping frequency from 1 Hz to 100 Hz. Figures 7, 8 and 9 shows the forced vibration response. It is found from the simulations that for 4mm circular crack the resonance frequency of beam is found to be 35.482 Hz and 35.02 Hz and 34 Hz for 6mm and 8mm circular cracks. In the following section the experimental studies are performed for the beam dimensions as that for simulations.

5. Experimental Setup

(a)

(b)

Figure 10. Experimental setup

The experimental setup for the vibration analysis of the cantilever beam has been shown in the above mentioned Figure 10(a). Figure 10 (b) shows the detailed view near the vicinity of beam. The initial displacement was provided there in order to provide the initial excitation to the cantilever beam. After this initial excitation was provided by using data acquisition system from the computer and the response of the cantilever beam was measured. The same procedure was repeated for recording the system response.

Table 1 shows the material and geometric properties of the beam used for the experimentation. Figure 11 shows the homogeneous copper beam mounted on shaker.

Table 1: Specifications of Model

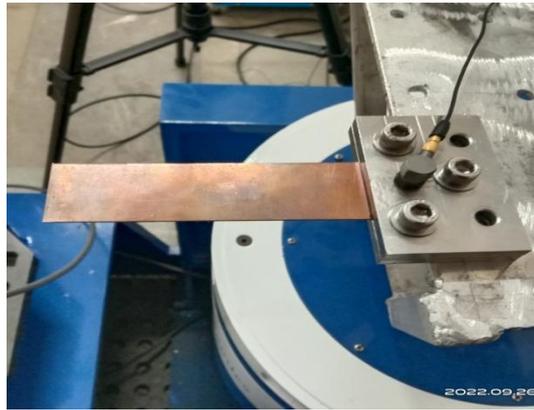| Material	Mass density (ρ) in kg/m^3	Length of the beam(l) in m	Width of the beam (b) in m	Thickness of the beam(h) in m	Elastic Modulus of the material (E) in N/m^2
Copper	8960	0.09	0.0254	0.0005	1.26×10^{11}

Figure 11. Cantilever beam without crack.

6. Experimental Analysis of Cantilever Beam without Crack

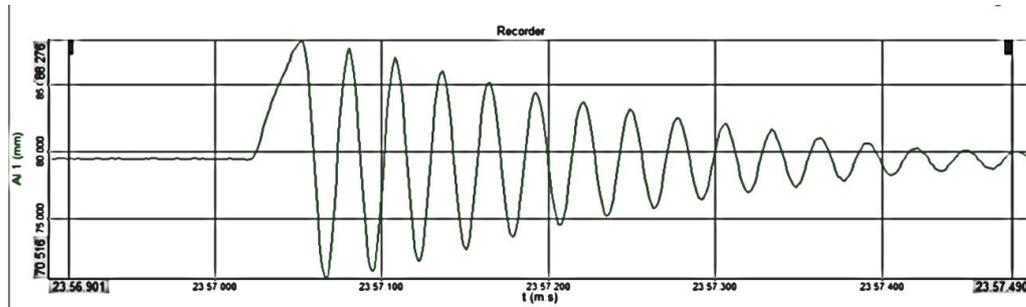### 6.1 Free vibration analysis

Figure 12. Time domain analysis of response of cantilever beam

Figure 12 shows the typical time-domain response of the beam under free vibration. The graph shows the displacement of the beam over time, with decay in amplitude due to the damping properties of the material. The measured natural frequency found to be 35.71 Hz.
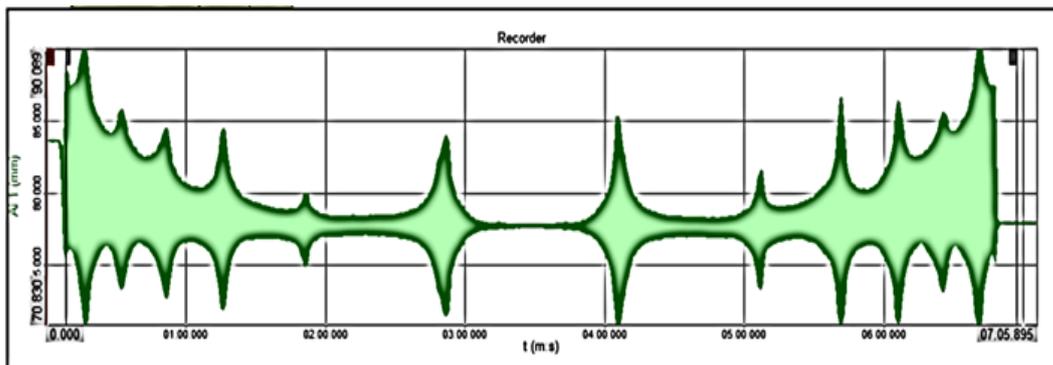
7.1 Forced Vibration Analysis

Figure 13. Forced vibration analysis in DEWESOFTX

The same copper beam is mounted on the shaker to perform forced vibration test. Figure 13 shows the time history response of the beam under forward sweep & reverse sweep.
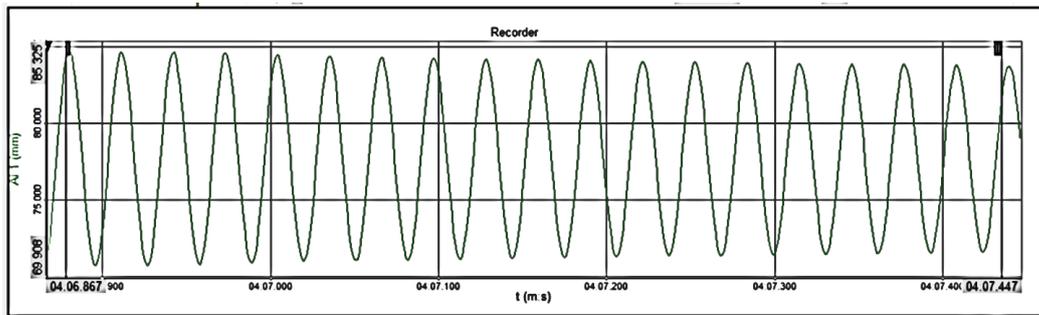
Figure 14. Time domain analysis of response of cantilever beam during forward sweep

The Figure 14 shows the magnified view of the response of the beam for the given input of 0.5 g acceleration by shaker. The amplitude of the vibration response increases as the frequency approaches the natural frequency, and then decreases as the frequency moves away from the peak. This behaviour is expected for a cantilever beam with a fixed end. Figure 15 shows the frequency response plot of the beam the natural frequency of the beam is found to be 36.11 Hz. The peak amplitude at resonance is found to be 4.78 mm. The measured natural frequency of beam with free and forced vibration tests are in good agreement.

Figure 15. Frequency response of forced vibration without crack
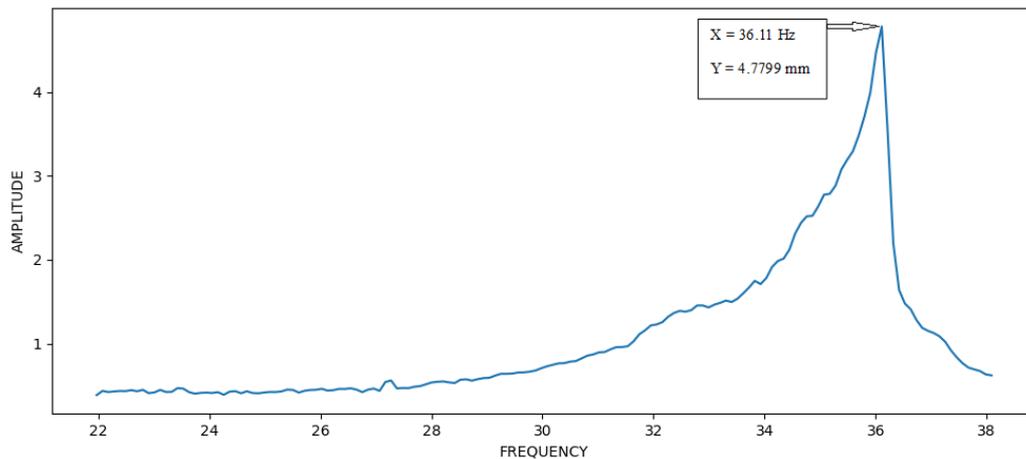
7. Experimental Analysis of Cantilever Beam with Crack

The same copper beam is used to perform forced vibration but with different sized circular shaped cracks at a distance of 20 mm from the fixed end. The following Figure 16 shows the typical view of the copper beam with circular crack mounted on the shaker.

(a)
Figure 16. Cantilever beam with crack.

In the following section, free and forced vibration tests are performed to measure the natural frequency of beam with different sized circular cracks.

7.1 Free Vibration Analysis

Table 2: Experimental results of free vibration with crack

Material	Circular crack size	Initial time	Final time	No. of cycles	Time period	Frequency
Copper Beam	4 mm	10.57	10.86	10	0.029	34.48 Hz
	6 mm	45.443	45.737	10	0.0294	34.01 Hz
	8 mm	48.562	48.868	10	0.0306	32.67 Hz

Table 2 shows the tabulated different size of the holes in beam and the corresponding measured natural frequency of the beam. For a circular crack size of 4 mm the natural frequency is measured as 34.48 Hz and is 34.01 Hz and 32.67 Hz for 6 mm and 8 mm size circular cracks respectively. It is observed from the results that with an increase in size of the circular crack, the natural frequency of the cantilever beam decrease. This may due to the loss of stiffness of the beam due to the circular crack [11].

7.2 Forced Vibration Analysis

The forced vibration tests are performed with base excitation acceleration of 0.5 g. The following Figures 17, 18 and 19 shows the displacement vs. frequency response plots. The measured peak amplitudes at resonance are 5.5mm, 5.41mm and 6.0mm for 4mm, 6mm and 8mm circular crack size copper beams respectively.
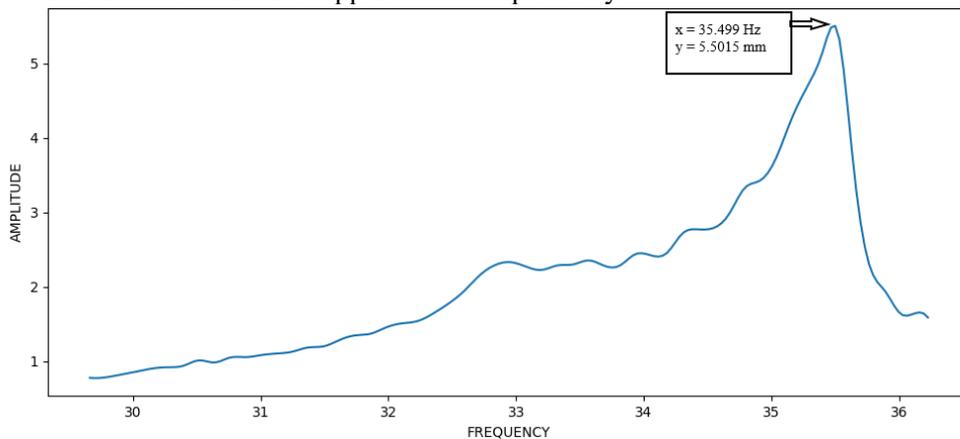

Figure 17. Frequency response of forced vibration with crack (4 mm circular crack)
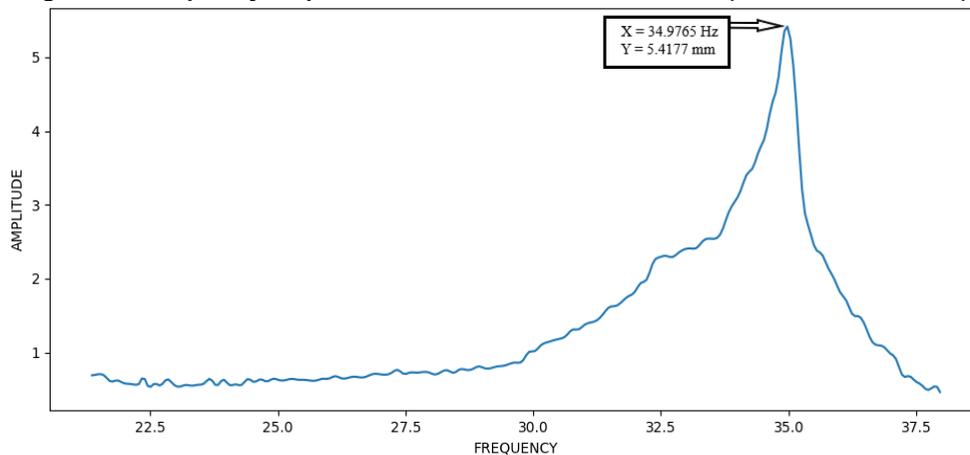

Figure 18. Frequency response of forced vibration with crack (6 mm circular crack)
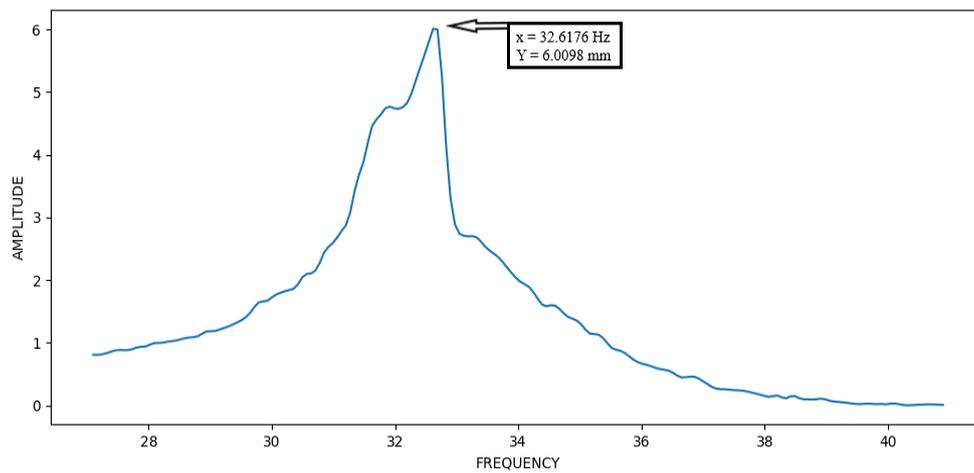
Figure 19. Frequency response of forced vibration with crack (8 mm circular crack)

8. Comparison of Results

Table 3: Comparison of experimental results of free and forced vibration analysis of cantilever beam without crack

Material	Experimental Results	
	Free Vibration Natural Frequency	Forced Vibration Natural Frequency
Copper Beam	35.71 Hz	36.11 Hz

Table 4: Comparison of experimental results of free and forced vibration analysis of cantilever beam with crack

Material	Circular crack Size	Experimental Results	
		Free Vibration Natural Frequency	Forced Vibration Natural Frequency
Copper Beam	4 mm	34.48 Hz	35.50 Hz
	6 mm	34.01 Hz	34.97 Hz
	8 mm	32.67 Hz	32.62 Hz

Table 3 shows the comparison of frequency results among the free vibration and forced vibration tests for the beam without crack, whereas, Table 4 shows the comparison of frequency results among the free vibration and forced vibration tests for the beam with crack. For a circular crack size of 4mm, the natural frequency with free vibration test is found to be 35.48 Hz and is 34.01 Hz and 32.67 Hz for the crack size of 6mm and 8mm. For a circular crack size of 4mm, the natural frequency with forced vibration test is found to be 35.50 Hz and is 34.97 Hz and 32.62 Hz for the crack size of 6mm and 8mm. It is observed from results they are in good agreement. In both cases, with increase in crack size the natural frequency of the cantilever beam decreases and trend is similar. This may be due to reduction in stiffness of the cantilever beam with increase in crack size.

As discussed, the natural frequency of cantilever beam without a crack is found to be 35.71 Hz for free vibration test and 36.11 Hz for forced vibration test. It is observed from results they are in good agreement.

9. Conclusions

An effort has been made to study the variation in the natural frequency of a machine component under homogeneous and cracked conditions. For this purpose, a cantilever beam was analyzed under dynamic loading conditions. Both free and forced vibration tests were performed, and the experimental results were validated using analytical and numerical simulations.

For the homogeneous copper beam (without crack), the natural frequencies obtained from the free vibration test were 37.9 Hz (analytical), 35.71 Hz (experimental), 35.753 Hz (numerical simulation), and 37.58 Hz (FEM analysis). The close agreement among these results confirms the reliability of the adopted methods. In the case of forced vibration, the natural frequencies were found to be 36.11 Hz (experimental) and 35.838 Hz (numerical simulation), again showing good consistency.

For the non-homogeneous condition (beam with a circular crack located 20 mm from the fixed end), a reduction in natural frequency was observed with an increase in crack size. Specifically, for a 4 mm circular crack, the natural frequencies were 34.48 Hz (free vibration) and 35.50 Hz (forced vibration); for a 6 mm crack, 34.01 Hz and 34.97 Hz; and for an 8 mm crack, 32.67 Hz and 32.62 Hz, respectively.

This consistent decrease in natural frequency with increasing crack size indicates that the presence and growth of cracks lead to a reduction in the stiffness of the beam. Hence, monitoring changes in natural frequency serves as an effective approach for identifying structural damage or defects in machine components.

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