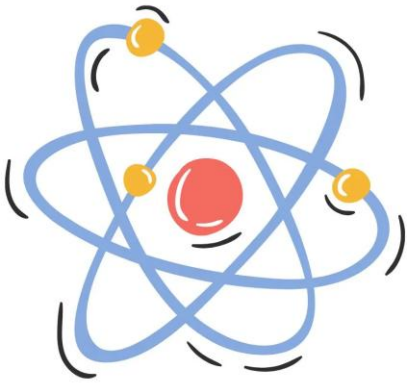

ISSN: 1672 - 6553

JOURNAL OF DYNAMICS AND CONTROL

VOLUME 9 ISSUE 11: P10-36

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JOURNAL OF DYNAMICS AND CONTROL
https://doi.org/10.71058/jodac.v9i11002

OPTIMIZATION MODELING OF DEA FOR MEASURING HOSPITAL PERFORMANCE EFFICIENCY WITH PYTHON CODING

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ABSTRACT: *Decision-making units (DMUs) are regarded in the standard DEA technique as entities that transform numerous inputs into multiple outputs so that their relative efficiency may be evaluated. The performance of hospital branches is assessed in this study using the CCR and BCC models, respectively, under constant and variable returns to scale. The paper's goal is twofold: Its objective is to evaluate the performance of seven branches of a hospital in a state Rajasthan. Evaluating the operational performance of hospitals has always been crucial because of their importance to the health and well-being of society. The primary goal is to assess the effectiveness of seven branches of Hospital locations in Rajasthan. The secondary goal is to demonstrate the applicability and effectiveness of Python-based Data Envelopment Analysis (DEA) using the CCR model for operational efficiency assessment. Hospital management will be able to examine and contrast operational efficacy across branches. The CCR and BCC models of Data Envelopment Analysis (DEA), which are implemented in Python, are used in the study to describe the effectiveness of these branches. Python offers strong modules and an adaptable syntax for Data Envelopment Analysis (DEA) that make data processing, visualization, and model implementation easier. It is ideal for both small- and large-scale efficiency assessments due to its adaptability and integration capabilities. To analysis the proposed method with computational calculation, a numerical example solves to validate the model. Finally, conclusion and future research directions are also presented.*

KEYWORDS: *Hospital efficiency; Data Envelopment Analysis (DEA); CCR model; Python*

Article Highlights:

1. DEA with hospitals' study addresses the increasing relevance of evaluating operational efficiency.
2. The importance of this research lies in its contribution to both policy-making and academic understanding by identifying operational benchmarks, highlighting inefficiencies, and offering insights into resource utilization in Hospitals.
3. Innovative use of PYTHON for modeling to measure operational efficiency and decision making,

1 INTRODUCTION

Hospitals are the frontline in the health sector, particularly in ensuring the well-being of the community.[1] They provides critical care, preventive services, and cutting-edge therapies, making them essential to preserving public health. In order to properly manage healthcare resources and enhance community well-being, they are essential. Assessing hospital efficiency becomes crucial as they manage scarce resources and rising demand in order to guarantee high-quality service delivery and efficient use of inputs. To assess the relative effectiveness of decision-making units (DMUs), including banks, hospitals, and educational institutions, a non-parametric technique known as Data Envelopment Analysis (DEA) is frequently employed. Charnes, Cooper, and Rhodes initially presented Data Envelopment Analysis (DEA) in 1978 with their groundbreaking CCR model. The basis for assessing the effectiveness of decision-making units

https://doi.org/10.71058/jodac.v9i11002

that use a variety of inputs and outputs was established by their work. By comparing several inputs and outputs without requiring a preset functional form, it offers a methodical framework for gauging efficiency. By creating an efficiency frontier, DEA finds the top-performing units and creates a standard by which others are evaluated. It is therefore a potent instrument for evaluating output, performance, and resource use. Because it takes into consideration the various resources, services, and results required in providing high-quality care, DEA is very useful in the healthcare industry. This allows for a fair assessment of hospital efficiency. Dr. Shiv Bhagwan Jhawar established Apex Hospitals in 1986 as SS Diagnostic Centre. Since then, it has expanded from a modest diagnostic facility to become one of Rajasthan's top multispecialty hospital chains. Its adventure in advanced healthcare delivery began in 1994 with the commissioning of the first 50-bed hospital. With the goal of offering top-notch, reasonably priced, and easily accessible healthcare, Apex Hospitals has grown to several cities, including Jaipur, Jhunjhunu, Bikaner, and Udaipur. It has established new standards in area healthcare over time by introducing cutting-edge services including robotic surgery, cancer, and minimally invasive surgery. Apex continues to fulfill its aim of providing patient-centered and compassionate care while upholding standards of quality thanks to its NABH and ISO accreditations. In order to assess these hospitals' effectiveness, the researcher will employ the Data Envelopment Analysis approach. The Envelopment Analysis approach (DEA) uses linear programming problem approaches (LPP) to evaluate and measure the efficacy of decision-making units at any kind of economic institution, including banks, colleges, hospitals, schools, and retail establishments. Only when the branches to be compared have comparable inputs and outputs is this feasible.

Assessing and contrasting the efficacy of organizational units, such as local government agencies, schools, colleges, hospitals, shops, bank branches, and other similar settings, with a relatively homogeneous collection of units is becoming increasingly important.

The usual measure of efficiency, i.e.:

$$Efficiency = \frac{Output}{Input}$$

1.1 Significance of DEA

Data Envelopment Analysis (DEA), which takes into account several inputs and outputs at once, is a potent method for assessing an organization's relative efficiency. In contrast to conventional single-ratio approaches, it offers an accurate evaluation of performance and identifies the highest-performing units that can be used as standards by others to implement successful practices. By identifying inefficiencies, DEA assists in resource optimization, assisting managers in more efficiently deploying personnel, funds, and equipment. Furthermore, it facilitates well-informed decision-making by providing practical insights for policy development and operational enhancements. Because of its adaptability, it may be used in a variety of industries, including manufacturing, banking, healthcare, and education, making it a highly useful technique for increasing organizational effectiveness.

1.2 DEA in Healthcare and Hospital Studies

Hospitals frequently employ Data Envelopment Analysis (DEA) to assess the efficacy and efficiency of different departments by contrasting the resources they use—such as doctors, nurses, beds, and equipment—with the results they attain, such as the number of patients treated and the rate of recovery. The most effective units are identified, enabling hospital management to

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implement best practices, reduce waste, and distribute resources as efficiently as possible. DEA promotes better patient care, expedites hospital operations, and raises the standard of services by making sure that staff, supplies, and prescriptions are used efficiently. It also offers useful information for strategic planning, such as staff management, technology investment, and departmental expansion, which helps hospitals effectively provide high-quality treatment.

1.3 Advantages and Limitations of DEA

When assessing organizational efficiency, DEA has a number of advantages. It is capable of managing several inputs and outputs at once, offering a thorough evaluation of performance. By pointing out inefficiencies, DEA makes it possible to make focused adjustments and identify the most effective units, which act as standards for others. Since it is non-parametric, it is adaptable and broadly applicable because it does not necessitate a predetermined functional relationship between inputs and outputs. DEA promotes ongoing development, aids in resource allocation optimization, and supports well-informed decision-making. It is a useful tool for improving organizational efficiency because of its adaptability, which enables application across a variety of industries, including healthcare, banking, education, and manufacturing.

Data Envelopment Analysis (DEA) has a number of drawbacks despite being a strong instrument for assessing effectiveness. Even the most effective units could not be fully efficient because DEA assesses relative efficiency rather than absolute efficiency. It lacks statistical tests to determine significance and is susceptible to outliers, which can skew results. The approach may miss qualitative elements like staff morale or patient satisfaction because it only considers the variables that are part of the model and depends on precise and comprehensive input-output data. Additionally, when working with huge datasets, models can become sophisticated and computationally demanding, and DEA just identifies inefficient units without elucidating the underlying causes. These restrictions emphasize that when using DEA, careful data selection and additional analysis are required.

1.4 How DEA Operates?

Decision-making units' (DMUs') efficiency is assessed using Data Envelopment Analysis (DEA), which compares the resources they use—such as personnel, equipment, and budget—with the outcomes they generate—such as patients treated, services rendered, or recovery rates. For each DMU, the initial step is to choose appropriate inputs and outputs. In order to identify the most effective units, DEA then creates an efficiency frontier. Units on the frontier are regarded as totally efficient, while those below it are regarded as inefficient. Every DMU is given an efficiency score that represents its relative performance; these scores usually range from 0 to 1. The comparison of inefficient and efficient units establishes benchmarks and improvement goals, demonstrating how efficiency can be attained by reducing inputs or increasing outputs. In general, DEA offers managers and legislators practical insights to improve operational performance, allocate resources optimally, and encourage ongoing development.

1.5 DEA, why?

Data Envelopment Analysis (DEA) is employed because it offers a potent method of calculating the relative efficiency of similar units by taking into account a number of inputs, including personnel, equipment, and financial resources, as well as several outputs, like services rendered or patients treated. In contrast to conventional ratio approaches, it pinpoints the top-performing units

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and positions them on an efficiency frontier so that they can be used as standards by others. In addition, DEA identifies inefficiencies and makes recommendations for how underperforming units might boost outputs or utilize less resources in order to become more efficient. This helps businesses increase production, reduce waste, and allocate resources as efficiently as possible. DEA helps managers and legislators make well-informed decisions by providing efficiency scores and specific improvement goals. Because it may be used in a variety of industries, such as healthcare, banking, education, and manufacturing, its adaptability further enhances its significance and makes it a highly useful instrument for performance assessment and enhancement.

1.5 Decision Making Units (DMUs)

Decision Making Units (DMUs) are the entities being assessed for efficiency in Data Envelopment Analysis (DEA). Each DMU uses specific inputs (such as personnel, expenses, money, or resources) to generate specific outputs (such as services, profits, or patient recoveries). To determine which DMUs are wasteful and which are efficient—those that fall on the efficiency frontier—DEA compares a number of DMUs. Hospitals or their divisions, bank branches, educational institutions, and business entities such as stores or companies are examples of DMUs. A DMU must function similarly, have comparable quantifiable inputs and outputs, make decisions on its own, and have a dataset with enough DMUs for meaningful comparison in order to be considered legitimate in DEA.

Mathematically, a DMU j is represented as $DMU_j = (x_{1j}, x_{2j}, \dots\dots, x_{mj}; y_{1j}, y_{2j}, \dots\dots, y_{sj})$, where x denotes inputs and y denotes outputs, thus capturing its role as a unit that transforms inputs into outputs for efficiency analysis.

1.6 Slack variable

A slack variable in Data Envelopment Analysis (DEA) is the extra input or output deficiency that keeps a Decision Making Unit (DMU) from operating at its highest level of efficiency. Slack in input indicates how much input can be decreased without affecting output, and slack in output indicates how much output can be raised without requiring additional inputs. There may still be gaps, or untapped potential, in a DMU with an efficiency score of 1. For instance, to maintain its production, a hospital could employ fewer physicians or serve more patients with the same resources. Only when all slack variables are zero and the efficiency score is one is a DMU deemed fully efficient.

1.7 Reduced Cost

Reduced cost in Data Envelopment Analysis (DEA) quantifies the amount that an inactive input or output needs to alter in order to have an impact on a DMU's efficiency score. It highlights factors that aren't currently affecting efficiency and illustrates how susceptible the model is to them. While a zero number implies the variable already helps to efficiency, a non-zero decreased cost suggests room for improvement. If a hospital's "administrative staff" input has zero weight, for example, its decreased cost demonstrates how much its importance must rise in order to affect total efficiency. Therefore, lower expenses aid in identifying elements that, with modification, could improve a DMU's performance.

1.8 Surplus Variable

A Decision Making Unit's (DMU) efficiency is evaluated using a number of indicators in Data Envelopment Analysis (DEA). Input excesses or output deficits are measured by slack variables,

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and a DMU is considered fully efficient only when its efficiency score is equal to one and all slacks are zero. Surplus variables record outputs that surpass efficiency goals, whereas reduced cost indicates the amount that an inactive input or output has to change in order to affect efficiency. When taken as a whole, these metrics offer a thorough picture of performance, pointing out areas that require development and resource efficiency.

1.9 Input-Oriented Model

In Data Envelopment Analysis (DEA), an input-oriented model aims to maintain consistent output levels while reducing the inputs a Decision Making Unit (DMU) uses. It determines how much input can be decreased without hurting output, which is very helpful when resources are expensive or scarce. If a DMU's inputs can be proportionally lowered, it is inefficient; if no input can be reduced without lowering output, it is efficient. One way to assess whether staffing levels can be lowered without compromising care is to look at a hospital department that treats 100 patients and has 10 doctors and 5 nurses. Managers can enhance operational efficiency and optimize resource utilization with this method.

1.10 Output-Oriented Model

An output-oriented model in Data Envelopment Analysis (DEA) aims to maximize a Decision Making Unit's (DMU) outputs while maintaining constant inputs. Since it establishes how much output can be increased without requiring more inputs, it is especially helpful when resources are fixed. A DMU is considered inefficient if proportionate output growth can be achieved with the same resources, and efficient if outputs cannot be increased without increasing inputs. For instance, with the same number of physicians and nurses, a hospital can determine how many more people it could treat. Input-oriented models that prioritize resource minimization are complemented by this model, which places an emphasis on output maximization and performance enhancement.

1.11 Efficiency Score

An efficiency score in Data Envelopment Analysis (DEA) measures a Decision Making Unit's (DMU) ability to convert inputs into outputs in comparison to its peers. Usually, scores fall between 0 and 1, where 1 denotes complete efficiency and lower numbers indicate inefficiency and potential for improvement. In order to determine how much outputs could be increased (output-oriented) or inputs reduced (input-oriented) to achieve optimum efficiency, DEA uses linear programming to generate this score. A hospital with a score of 0.85, for instance, might achieve complete efficiency by increasing outputs or cutting inputs by 15%. Managers can analyze performance, pinpoint areas for resource or output improvement, and identify high-performing units with the aid of efficiency scores.

1.12 BCC Model

Banker, Charnes, and Cooper (1984) developed the BCC model in Data Envelopment Analysis (DEA), which assesses Decision Making Unit (DMU) efficiency while accounting for variable returns to scale (VRS), which means that efficiency might change based on the number of activities. The BCC model can differentiate between inefficiency brought on by inefficient scale operations (scale inefficiency) and inefficiency resulting from poor management (pure technical inefficiency), in contrast to the CCR model, which makes the assumption that returns to scale are constant. It can be used in two ways: input-oriented, where the goal is to minimize inputs while

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maintaining outputs, or output-oriented, where the goal is to maximize outputs while maintaining inputs. Technical efficiency under VRS is the efficiency score that was obtained. It goes from 0 to 1, where 1 denotes complete efficiency. For example, the BCC model can identify if a department's inefficiency in a hospital is due to poor resource management or to the department's size. Overall, by taking into consideration the effects of scale, the BCC model offers a flexible and perceptive assessment of efficiency.

1.13 CCR Model

Charnes, Cooper, and Rhodes (1978) developed the CCR model in Data Envelopment Analysis (DEA), which assesses the effectiveness of Decision Making Units (DMUs) under the premise of constant returns to scale (CRS), which states that any proportionate change in inputs causes an equivalent change in outputs. It can be used in two ways: input-oriented, where the goal is to decrease inputs while maintaining constant outputs, or output-oriented, where the goal is to increase outputs while maintaining the same level of inputs. With a score of 1 denoting complete efficiency and a value below 1 denoting inefficiency in comparison to the top-performing DMUs, the model offers an overall technical efficiency score. The CCR model directs less efficient units on how much to increase outputs or decrease inputs to enhance performance and assists in identifying efficient DMUs as benchmarks. However, unlike the BCC model, which takes varying returns to scale into consideration, it does not distinguish between technical and scale inefficiencies because it assumes constant returns to scale.

1.14 Basic Efficiency

Basic efficiency is the primary metric used in Data Envelopment Analysis (DEA) to assess how successfully a Decision Making Unit (DMU) transforms inputs into outputs in comparison to other units. It is expressed as the ratio of the weighted sum of outputs to the weighted sum of inputs, given by the formulation:

$$\boldsymbol{E = \frac{\sum_{r=0}^{s} u_r y_{ro}}{\sum_{i=0}^{m} v_i x_{io}}}$$

where,

E = efficiency score of the DMU under evaluation
y_{ro} = amount of output r produced by the DMU
x_{io} = amount of input i used by the DMU
u_r = weight assigned to output r
v_i = weight assigned to input i
s = number of outputs
m = number of inputs

The efficiency score goes from 0 to 1, with 1 denoting complete efficiency (the DMU is on the efficiency frontier) and a score below 1 denoting inefficiency, indicating that the DMU could become more efficient by either increasing its outputs (output-oriented model) or decreasing its inputs (input-oriented model). In DEA, this fundamental efficiency metric is used as the standard to determine which DMUs are performing the best and to direct inefficient ones to increase output production or resource use.

1.15 Price Efficiency (PE)

https://doi.org/10.71058/jodac.v9i11002

Price efficiency, also known as allocative efficiency, is a metric used in Data Envelopment Analysis (DEA) to determine if a Decision Making Unit (DMU) is using its inputs in the most cost-effective proportions given their costs. By taking into account not just the amount of input used but also whether the selected input mix lowers overall cost, it goes beyond technical efficiency. Price efficiency is formulated as

$$PE = \frac{w^T x^*}{w^T x}$$

where w is the vector of input prices, x is the actual input vector used, x^* is the cost-minimizing input vector, $w^T x$ is the actual cost, and $w^T x^*$ is the minimum possible cost. The efficiency score ranges between 0 and 1, with PE = 1 indicating full price efficiency (optimal input mix) and PE < 1 indicating that the DMU could reduce costs by adjusting its input proportions without reducing output. Together with technical efficiency (TE), price efficiency determines cost efficiency (CE = TE × PE), helping managers identify whether inefficiency arises from poor resource use, suboptimal input choice, or both.

1.16 Relative Efficiency (RE)

Relative efficiency in Data Envelopment Analysis (DEA) quantifies a Decision Making Unit's (DMU) performance in relation to other DMUs in the dataset, as opposed to against a strict benchmark. A DMU is said to be reasonably efficient if it falls on the efficiency frontier and no other DMU or set of DMUs can provide more outputs with the same inputs or the same outputs with fewer inputs. Relative efficiency is calculated as the ratio of weighted outputs to weighted inputs:

$$RE = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

where $y_{ro}$and x_{io} are the outputs and inputs of the DMU, u_r and v_i are their respective weights, and s and m are the number of outputs and inputs. The score ranges from 0 to 1, with RE = 1 indicating that the DMU is fully efficient and lies on the frontier, while RE < 1 shows relative inefficiency, meaning there is potential to improve by reducing inputs or increasing outputs. Relative efficiency serves as the foundation of DEA, helping managers benchmark DMUs, identify best practices, and guide inefficient units toward better performance.

1.17 Python

Data Envelopment Analysis (DEA) uses Python, a strong and flexible programming language, to develop and evaluate the effectiveness of Decision Making Units (DMUs). Pyomo and CVXPY are optimization libraries that help define and solve the linear programming problems that underlie DEA models, whereas Pandas and NumPy are libraries for organizing, cleaning, and manipulating input-output data. Researchers can automate the computation of technical, cost, and price efficiency ratings for several DMUs and apply CCR, BCC, input-oriented, and output-oriented models using Python. Visualization tools such as Matplotlib and Seaborn also make it possible to plot efficiency boundaries, rankings, and comparisons, which facilitates the interpretation of results. In industries like healthcare, finance, and manufacturing, Python is the perfect tool for thorough efficiency evaluation because of its scalability, which enables it to handle enormous

https://doi.org/10.71058/jodac.v9i11002

datasets efficiently, and its flexibility, which facilitates integration with other data analysis, machine learning, or optimization tasks.

2 Literature Review

This review of the literature focuses on studies that use Python to implement Data Envelopment Analysis (DEA), which is a technique used to assess hospital efficiency. Hospitals are vital to the delivery of healthcare, and evaluating their operational effectiveness is critical to optimizing resource allocation and enhancing patient outcomes. Recent studies demonstrate how Python is increasingly being used for DEA because of its adaptability, repeatability, and capacity to manage intricate information and produce perceptive visuals. The performance of decision-making units (DMUs) across a variety of inputs and outputs is systematically evaluated using Data Envelopment Analysis (DEA), which has been frequently used to evaluate hospital efficiency. The foundation for efficiency analysis in organizational contexts was laid by Rizwanullah (2016), who showed how to analyze performance and pinpoint areas for improvement using DEA (CCR Model) backed by optimization tools. To classify hospitals in Indonesia and Malaysia, Maulana et al. (2024) used DEA in conjunction with cluster analysis. This demonstrated how combining DEA with cutting-edge analytical methods can yield lucid insights on hospital performance trends. By successfully managing uncertainty in hospital data and emphasizing methods to increase overall efficiency, Omrani et al. (2018) further improved DEA applications in the healthcare industry by combining fuzzy clustering and cooperative game theory.

The techniques developed by Andersen and Petersen (1993) to rank DEA's efficient units are especially helpful in differentiating between hospitals with comparable efficiency scores. Prior hospital-focused DEA research by Chilingerian (1995) and Ozcan et al. (1992) examined the relationship between physician efficiency and hospital ownership, highlighting the relevance of DEA in the healthcare industry. Python (Python.org, 2022) and Jupyter Notebook (Jupyter.org, 2022) are being used more in modern implementations because they offer adaptable, interactive, and reproducible platforms for DEA computation and visualization. Tone et al. (2020) discussed methodological improvements such managing negative data in DEA models, whereas Cai et al. (2005, 2006) showed that Python is appropriate for intricate scientific calculations. All these studies demonstrate that Python-based DEA is a reliable, useful, and effective method for evaluating hospital performance, facilitating improved benchmarking, resource allocation, and well-informed decision-making in healthcare environments. The DEA approach is based on efficiency analysis, which is a theoretical framework for assessing trade-offs between different inputs and outcomes. Keeney and Raiffa (1993) highlighted decision-making with numerous objectives. The method's versatility in evaluating performance across enterprises was demonstrated by Ramadan (2016)'s application of DEA to the banking industry. This approach can be expanded to healthcare settings for hospital efficiency evaluation. Sreedevi (2016) demonstrated the useful application of DEA models in actual operational evaluations by using the CCR model in DEA to gauge private bank efficiency. Kumari (2020) examined a number of DEA-based techniques for gauging the effectiveness of decision-making units, highlighting the adaptability and methodological soundness of DEA in evaluating performance in a variety of industries, including healthcare facilities.

The literature highlights how crucial it is to use Data Envelopment Analysis (DEA) to assess hospital efficiency, especially when Python is used. Hospital efficiency is multifaceted by nature and depends on several variables, including patient load, staff performance, resource allocation,

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and operational rules. According to recent research, Python-based DEA makes analysis flexible and repeatable, enabling researchers to work with complicated datasets, complete calculations quickly, and produce perceptive visuals. Comparative studies of hospitals in various geographical areas reveal differences in effectiveness and pinpoint optimal procedures that might guide healthcare administration and policy. All things considered, DEA applied with Python has shown itself to be a useful tool for evaluating hospital performance, identifying critical aspects of efficiency, and assisting healthcare organizations in making strategic decisions. The actual use of DEA in hospital efficiency studies is improved by Python's data processing, modeling, and visualization capabilities.

3 Problem Formulation

Data and statistics from the National Health Profile (NHP) website and Hospital reports on respective websites were used to select the study sample. The study sample comprised seven Rajasthan-based hospitals. Because hospitals are Healthcare institutions that offer a variety of products, it is challenging to handle all of their inputs and outputs. As a result, the variables included in this study will only be those that accurately represent the primary activities of hospitals. The variables of the model determined for the current study includes.

1. *Outputs:* The following three primary outputs were identified and chosen to serve as the primary inputs for hospitals' primary operations and to boost their profits:

    a. Number of patients treated per month under 14 (in Hundreds): This output measures the total number of pediatric patients (under 14 years) treated in the hospital during a month. It reflects the hospital's efficiency in providing medical care to children, indicating its capacity to manage pediatric healthcare services effectively and meet the treatment needs of younger patients.

    b. Number of patients treated per month between 14 and 65 (in Hundreds): This output measures the total number of adult patients, aged between 14 and 65 years, treated in the hospital each month. It reflects the hospital's efficiency in managing and delivering healthcare services to the adult population, indicating its ability to handle patient volume effectively and provide timely medical care to meet the healthcare needs of working-age patients.

    c. Number of patients treated per month above 65 (in Hundreds): This output measures the total number of senior patients, aged above 65 years, treated in the hospital each month. It reflects the hospital's efficiency in providing medical care to the elderly population, indicating its ability to manage higher-risk and age-sensitive cases effectively and ensure timely treatment for older patients.

2. *Inputs:* Two key variables reflects the main activity of hospitals and have an impact on hospitals' outputs and as follows:

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a. Capital(number of beds): This input represents the hospital's capital resources in terms of the total number of beds available. It reflects the hospital's capacity to accommodate and treat patients, serving as a measure of its infrastructure. A higher number of beds indicates greater ability to provide healthcare services, and in DEA, it is used to assess how efficiently the hospital utilizes this capital to produce healthcare outputs.

b. Labor(in thousands): This input represents the total labor hours used by the hospital during one month, including doctors, nurses, ward staff, pharmacists, and administrative personnel. It reflects the human resource effort required to provide healthcare services. In DEA, this input is used to evaluate how efficiently the hospital utilizes its workforce to produce outputs such as treated patients, surgeries, and other medical services. Higher efficiency indicates that the hospital can achieve better healthcare outcomes with the available labor hours.

4. Mathematical Modelling and Analysis

Let's consider a group of hospitals. Let's look at a collection of seven branches of tiny healthcare institutions. For the sake of simplicity, let us assume that every hospital "converts" three distinct outputs from two inputs.

The two inputs that every hospital uses are:

Input 1 = Capital (number of beds)
Input 2 = labor (in ,000, hours)

The outputs produced by each hospital are:

Output 1 = Number of patients treated per month under 14 (in Hundreds)
Output 2 = Number of patients treated per month between 14 and 65 (in Hundreds)
Output 3 = Number of patients treated per month above 65 (in Hundreds)

Suppose that inputs and outputs for seven hospitals are as given in table 1. To determine whether a hospital is efficient. Let's define t_r = value of one unit of output r and w_s = cost of one unit of input s. The $efficiency$ of hospital i is defined to be.

$$\frac{value\ of\ hospital\ i's\ outputs}{cost\ of\ hospital\ i's\ inputs}$$

For the data in table 1, it is find the efficiency of each hospital to be follows:

$$\text{Hospital H1 efficiency} = \frac{19.89t_1+39.7t_2+6.63t_3}{250w_1+42.436w_2}$$

$$\text{Hospital H2 efficiency} = \frac{2.83t_1+7.65t_2+1.28t_3}{50w_1+16.686w_2}$$

$$\text{Hospital H3 efficiency} = \frac{7.65t_1+15.3t_2+2.55t_3}{100w_1+18.952w_2}$$

https://doi.org/10.71058/jodac.v9i11002

$$\text{Hospital H4 efficiency} = \frac{4.59t_1+9.18t_2+1.53t_3}{60w_1+11.948w_2}$$

$$\text{Hospital H5 efficiency} = \frac{7.65t_1+15.3t_2+2.55t_3}{100w_1+17.304w_2}$$

$$\text{Hospital H6 efficiency} = \frac{7.65t_1+15.3t_2+2.55t_3}{100w_1+17.716w_2}$$

$$\text{Hospital H7 efficiency} = \frac{15.3t_1+30.6t_2+5.1t_3}{200w_1+40.376w_2}$$

Table 1 Inputs and Outputs for hospital

HOSPITALS	INPUTS		OUTPUTS		
	$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$	β_1	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$
H1	250	42.436	19.89	39.78	6.63
H2	50	16.686	3.83	7.65	1.28
H3	100	18.952	7.65	15.3	2.55
H4	60	11.948	4.59	9.18	1.53
H5	100	17.304	7.65	15.3	2.55
H6	100	17.716	7.65	15.3	2.55
H7	200	40.376	115.3	30.6	5.1

The approach of DEA practices the given four ideas to determine if hospital is efficient.

1. No hospital is thought to be more efficient than 100%. Each hospitals efficiency must therefore be less than or equal to 1. For hospital H1 , we find that $(19.89t_1 + 39.78t_2 + 6.63t_3)/(250w_1 + 42.436w_2) \leq 1$. Multiplying both sides of this inequality by $(250w_1 + 42.436w_2)$ yields the LP constraint $250w_1 + 42.436w_2 - 19.89t_1 - 39.78t_2 - 6.63t_3 \geq 0$.
2. Lets we are attentive in assessing the efficiency of hospital i. We make an effort to select output prices $(t_1, t_2\ and\ t_3)$ and input cost $(w_1 and\ w_2)$ that maximize efficiency. If hospital $i's$ efficiency is one, it is deemed efficient; if it is less than one, it is deemed inefficient.
3. We could scale the output prices to make the cost of Hospital $i's$ inputs equal one in order to make calculations easier. Hence, for Hospital H2 add the constraint $50w_1 + 16.686w_2 = 1$.
4. Every input cost and output price must be strictly positive. if, for example, $t_i = 0$, then DEA could not detect an efficiency involving output i; if $w_j = 0$, DEA will not detect an efficiency involving input j.
5. Point (1)-(4) lead to the following linear programming for testing the efficiency of each Branch of Hospitals.

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HOSPITAL H1 LP

$$\boldsymbol{max\ z} = 19.89t_1 + 39.78t_2 + 6.63t_3 \quad (1)$$

$$\boldsymbol{s.t.}$$

$$-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$250w_1 + 42.436w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$
$$w_2 \geq 0.0001 \quad (14)$$

HOSPITAL H2 LP

$$\boldsymbol{max\ z} = 3.83t_1 + 7.65t_2 + 1.28t_3 \quad (1)$$

$$\boldsymbol{s.t.}$$

$$-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$50w_1 + 16.686w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$
$$w_2 \geq 0.0001 \quad (14)$$

HOSPITAL H3 LP

$$\boldsymbol{max\ z} = 7.65t_1 + 15.3t_2 + 2.55t_3 \quad (1)$$

$$\boldsymbol{s.t.}$$

$$-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$100w_1 + 18.952w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$
$$w_2 \geq 0.0001 \quad (14)$$

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HOSPITAL H4 LP

$$\boldsymbol{max}\ \boldsymbol{z} = 4.59t_1 + 9.18t_2 + 1.53t_3 \quad (1)$$

$$\boldsymbol{s.t.}\quad -19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$60w_1 + 11.948w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$
$$w_2 \geq 0.0001 \quad (14)$$

HOSPITAL H5 LP

$$\boldsymbol{max}\ \boldsymbol{z} = 7.65t_1 + 15.3t_2 + 2.55t_3 \quad (1)$$

$$\boldsymbol{s.t.}\quad -19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$100w_1 + 17.304w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$
$$w_2 \geq 0.0001 \quad (14)$$

HOSPITAL H6 LP

$$\boldsymbol{max}\ \boldsymbol{z} = 7.65t_1 + 15.3t_2 + 2.55t_3 \quad (1)$$

$$\boldsymbol{s.t.}\quad -19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$100w_1 + 17.716w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$

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$$w_2 \geq 0.0001 \quad (14)$$

HOSPITAL H7 LP $\boldsymbol{max\ z} = 15.3t_1 + 30.6t_2 + 5.1t_3$ (1)

$\boldsymbol{s.t.}$

$$-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 \geq 0 \quad (2)$$
$$-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 \geq 0 \quad (3)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 \geq 0 \quad (4)$$
$$-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 \geq 0 \quad (5)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 \geq 0 \quad (6)$$
$$-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 \geq 0 \quad (7)$$
$$-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 \geq 0 \quad (8)$$
$$200w_1 + 40.376w_2 = 1 \quad (9)$$
$$t_1 \geq 0.0001 \quad (10)$$
$$t_2 \geq 0.0001 \quad (11)$$
$$t_3 \geq 0.0001 \quad (12)$$
$$w_1 \geq 0.0001 \quad (13)$$
$$w_2 \geq 0.0001 \quad (14)$$

From the value of optimal objective function of each LP obtained from PYTHON programming using DEA model, we find that:

Table 2 Efficiency calculation

Banks	Efficiency	Banks	Efficiency
H1	**1.0000**	**H5**	**0.9615**
H2	**0.9645**	**H6**	**0.9615**
H3	**0.9613**	**H7**	**0.9609**
H4	**0.9614**		

5. PYTHON program (Hospital-WISE)

HOSPITAL H1 LP :

MAX $19.89t_1 + 39.78t_2 + 6.63t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$

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8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$
11) $t_3 >= 0$
12) $w_1 >= 0$
13) $w_2 >= 0$
14) $250w_1 + 42.436w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 1.0000

Variables Table:

Variable	Value	Reduced Cost
t1	0.001784	0.0
t2	0.000100	0.0
t3	0.144876	0.0
w1	0.000100	0.0
w2	0.022976	0.0

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	-0.000000e+00	-1.0
C2	-1.953328e-01	-0.0
C3	-6.082151e-02	-0.0
C4	-4.974533e-02	-0.0
C5	-2.295743e-02	-0.0
C6	-3.242345e-02	-0.0
C7	-1.110223e-16	-0.0
C8	-0.000000e+00	1.0

HOSPITAL H2 LP :

$MAX\ 3.83t_1 + 7.65t_2 + 1.28t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65\,t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$
8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$

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11) $t_3 >= 0$
12) $w_1 >= 0$
13) $w_2 >= 0$
14) $50w_1 + 16.686w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 0.9645

Variables Table:

Variable	Value	Reduced Cost
t1	0.000100	-1.000000e-02
t2	0.000100	-3.000000e-02
t3	0.752629	-2.220446e-16
w1	0.019967	-7.105427e-15
w2	0.000100	-7.914377e+00

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	-3.191891e-16	-0.193062
C2	-3.548624e-02	-0.000000
C3	-7.705777e-02	-0.000000
C4	-4.629234e-02	-0.000000
C5	-7.689297e-02	-0.000000
C6	-7.693417e-02	-0.000000
C7	-1.443627e-01	-0.000000
C8	-0.000000e+00	0.965309

HOSPITAL H3 LP :

$MAX\ 7.65t_1 + 15.3t_2 + 2.55t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$
8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$

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11) $t_3 >= 0$
12) $w_1 >= 0$
13) $w_2 >= 0$
14) $100w_1 + 18.952w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 0.9613

Variables Table:

Variable	Value	Reduced Cost
t1	0.000100	8.881784e-16
t2	0.000100	1.776357e-15
t3	0.376099	0.000000e+00
w1	0.009981	0.000000e+00
w2	0.000100	-1.901539e+00

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	1.249001e-16	-0.384615
C2	-1.816585e-02	-0.000000
C3	-3.865169e-02	-0.000000
C4	-2.324869e-02	-0.000000
C5	-3.848689e-02	-0.000000
C6	-3.852809e-02	-0.000000
C7	-6.755058e-02	-0.000000
C8	-0.000000e+00	0.961538

HOSPITAL H4 LP:

$MAX\ 4.59t_1 + 9.18t_2 + 1.53t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$
8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$

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11) $t_3 >= 0$
12) $w_1 >= 0$
13) $w_2 >= 0$
14) $60w_1 + 11.948w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 0.9614

Variables Table:

Variable	Value	Reduced Cost
t1	0.001288	-8.881784e-16
t2	0.000100	-1.776357e-15
t3	0.623881	0.000000e+00
w1	0.016647	1.421086e-14
w2	0.000100	-1.695539e+00

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	5.698567e-16	-0.230769
C2	-2.973966e-02	-0.000000
C3	-6.428902e-02	-0.000000
C4	-3.863109e-02	-0.000000
C5	-6.412422e-02	-0.000000
C6	-6.416542e-02	-0.000000
C7	6.409803e-16	-0.000000
C8	-0.000000e+00	0.961538

HOSPITAL H5 LP:

$MAX\ \ 7.65t_1 + 15.3t_2 + 2.55t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$
8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$

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11) $t_3 >= 0$
12) $w_1 >= 0$
13) $w_2 >= 0$
14) $100w_1 + 17.304w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 0.9615

Variables Table:

Variable	Value	Reduced Cost
t1	0.000776	8.881784e-16
t2	0.062439	1.776357e-15
t3	0.000100	0.000000e+00
w1	0.009983	1.421086e-14
w2	0.000100	-3.169231e-01

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	-3.191891e-16	-0.384615
C2	-2.004564e-02	-0.000000
C3	-3.865803e-02	-0.000000
C4	-2.325250e-02	-0.000000
C5	-3.849323e-02	-0.000000
C6	-3.853443e-02	-0.000000
C7	-2.480655e-16	-0.000000
C8	-0.000000e+00	0.961538

HOSPITAL H6 LP

$MAX\ \ 7.65t_1 + 15.3t_2 + 2.55t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65\,t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$
8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$

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11) $t_3 >= 0$
12) $w_1 >= 0$
13) $w_2 >= 0$
14) $100w_1 + 17.716w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 0.9615

Variables Table:

Variable	Value	Reduced Cost
t1	0.000776	8.881784e-16
t2	0.000100	1.776357e-15
t3	0.374119	0.000000e+00
w1	0.009982	-1.421086e-14
w2	0.000100	-7.130769e-01

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	1.240327e-16	-0.384615
C2	-1.817475e-02	-0.000000
C3	-3.865645e-02	-0.000000
C4	-2.325155e-02	-0.000000
C5	-3.849165e-02	-0.000000
C6	-3.853285e-02	-0.000000
C7	-2.489328e-16	-0.000000
C8	-0.000000e+00	0.961538

HOSPITAL H7 LP

$MAX\ \ 15.3t_1 + 30.6t_2 + 5.1t_3$
SUBJECT TO
2) $-19.89t_1 - 39.78t_2 - 6.63t_3 + 250w_1 + 42.436w_2 >= 0$
3) $-3.83t_1 - 7.65\,t_2 - 1.28t_3 + 50w_1 + 16.686w_2 >= 0$
4) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 18.952w_2 >= 0$
5) $-4.59t_1 - 9.18t_2 - 1.53t_3 + 60w_1 + 11.948w_2 >= 0$
6) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.304w_2 >= 0$
7) $-7.65t_1 - 15.3t_2 - 2.55t_3 + 100w_1 + 17.716w_2 >= 0$
8) $-115.3t_1 - 30.6t_2 - 5.1t_3 + 200w_1 + 40.376w_2 >= 0$
9) $t_1 >= 0$
10) $t_2 >= 0$
11) $t_3 >= 0$

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12) $w_1 >= 0$
13) $w_2 >= 0$
14) $200w_1 + 40.376w_2 = 1$

LP Optimum found: Optimal
Objective Function Value (Z): 0.9609

Variables Table:

Variable	Value	Reduced Cost
t1	0.000100	0.000000e+00
t2	0.000100	0.000000e+00
t3	0.187516	-8.881784e-16
w1	0.004980	0.000000e+00
w2	0.000100	-6.180000e+00

Constraints Table:

Constraint	Slack/Surplus	Dual Price
C1	1.249001e-16	-0.769231
C2	-9.491006e-03	-0.000000
C3	-1.941617e-02	-0.000000
C4	-1.170738e-02	-0.000000
C5	-1.925137e-02	-0.000000
C6	-1.929257e-02	-0.000000
C7	-2.907954e-02	-0.000000
C8	-0.000000e+00	0.961538

6. Result Analysis

Dual Price and DEA

The DUAL PRICES section of PYTHON's outputs provides valuable information about the inefficiencies of hospital H2, H3, H4 ,H5 ,H6 and H7. Suppose all hospitals their efficiency constraints have non-zero dual prices in hospitals H2, H3, H4 , H5 , H6 and H7 LP. If we calculate mean of output vectors and input vectors for these hospitals, we obtain the following:

Averaged output vector [Hospital H2]

$$0.193062\begin{bmatrix}19.89\\39.78\\6.63\end{bmatrix} = \begin{bmatrix}3.84\\7.68\\1.2800011\end{bmatrix}$$

Averaged input vector [Hospital H2]

https://doi.org/10.71058/jodac.v9i11002

$$0.193062\begin{bmatrix}250\\42.436\end{bmatrix}=\begin{bmatrix}48.2655\\8.193\end{bmatrix}$$

Suppose we create a composite hospital by combining 0.193062 of hospital H1 with 0 of hospitals (H2,H3,H4,H5,H6,H7). The averaged output vector tells us that the composite hospital produces 3.84-3.83=0.01 more of output 1, 7.68-7.65=0.03 more of output 2 and 1.2800011-1.28=0.0000011 more output 3.

From the averaged input vector for the composition hospital, we find that the composition hospital uses 48.2655-50= -1.7345 less input 1, 8.193-16.686= -8.493 less input 2. We know see exactly where hospital H2 is inefficient . By the way, the objective function value of 0.9645 for hospital H2's LP implies that the more efficient composite hospital produces its superior outputs by using at most 96.45% as much of each input.

Averaged output vector [Hospital H3]

$$0.384615\begin{bmatrix}19.89\\39.78\\6.63\end{bmatrix}=\begin{bmatrix}7.649\\15.299\\2.549\end{bmatrix}$$

Averaged input vector [Hospital H3]

$$0.384615\begin{bmatrix}250\\42.436\end{bmatrix}=\begin{bmatrix}96.15\\16.323\end{bmatrix}$$

Suppose we create a composite hospital by combining 0.384615 of hospital H1 with 0 of hospitals (H2,H3,H4,H5,H6,H7). The averaged output vector tells us that the composite hospital produces 7.649-7.65= -0.001 less of output 1, 15.299-15.33= -0.001 less of output 2 and 2.549-2.55= -0.001 less output 3.

From the averaged input vector for the composition hospital, we find that the composition hospital uses 96.15-100= -3.85 less input 1, 16.323-18.952= -2.629 less input 2. We know see exactly where hospital H3 is inefficient . By the way, the objective function value of 0.9613 for hospital H3's LP implies that the more efficient composite hospital produces its superior outputs by using at most 96.13% as much of each input.

Averaged output vector [Hospital H4]

$$0.230769\begin{bmatrix}19.89\\39.78\\6.63\end{bmatrix}=\begin{bmatrix}4.589\\9.179\\1.529\end{bmatrix}$$

Averaged input vector [Hospital H4]

$$0.230769\begin{bmatrix}250\\42.436\end{bmatrix}=\begin{bmatrix}57.69\\9.79\end{bmatrix}$$

https://doi.org/10.71058/jodac.v9i11002

Suppose we create a composite hospital by combining 0.230769 of hospital H1 with 0 of hospitals (H2,H3,H4,H5,H6,H7). The averaged output vector tells us that the composite hospital produces 4.589-4.59= -0.001 less of output 1, 9.179-9.18= -0.001 less of output 2 and 1.529-1.53= -0.001 less output 3.

From the averaged input vector for the composition hospital, we find that the composition hospital uses 57.69-60= -2.31 less input 1, 9.79-11.948= -2.158 less input 2. We know see exactly where hospital H4 is inefficient . By the way, the objective function value of 0.9614 for hospital H4's LP implies that the more efficient composite hospital produces its superior outputs by using at most 96.14% as much of each input.

Averaged output vector [Hospital H5]

$$0.384615\begin{bmatrix}19.89\\39.78\\6.63\end{bmatrix}=\begin{bmatrix}7.649\\15.299\\2.549\end{bmatrix}$$

Averaged input vector [Hospital H5]

$$0.384615\begin{bmatrix}250\\42.436\end{bmatrix}=\begin{bmatrix}96.154\\16.32\end{bmatrix}$$

Suppose we create a composite hospital by combining 0.384615 of hospital H1 with 0 of hospitals (H2,H3,H4,H5,H6,H7). The averaged output vector tells us that the composite hospital produces 7.649-7.65= -0.001 less of output 1, 15.299-15.3= -0.001 less of output 2 and 2.549-2.55= -0.001 less output 3.

From the averaged input vector for the composition hospital, we find that the composition hospital uses 96.154-100= -3.846 less input 1, 16.32-17.304= -0.984 less input 2. We know see exactly where hospital H5 is inefficient . By the way, the objective function value of 0.9615 for hospital H5's LP implies that the more efficient composite hospital produces its superior outputs by using at most 96.15% as much of each input.

Averaged output vector [Hospital H6]

$$0.384615\begin{bmatrix}19.89\\39.78\\6.63\end{bmatrix}=\begin{bmatrix}7.649\\15.299\\2.549\end{bmatrix}$$

Averaged input vector [Hospital H6]

$$0.384615\begin{bmatrix}250\\42.436\end{bmatrix}=\begin{bmatrix}96.154\\16.32\end{bmatrix}$$

Suppose we create a composite hospital by combining 0.384615 of hospital H1 with 0 of hospitals (H2,H3,H4,H5,H6,H7). The averaged output vector tells us that the composite hospital produces

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7.649-7.65= -0.001 less of output 1, 15.299-15.3= -0.001 less of output 2 and 2.549-2.55= -0.001 less output 3.

From the averaged input vector for the composition hospital, we find that the composition hospital uses 96.154-100= -3.846 less input 1, 16.32-17.716= -1.396 less input 2. We know see exactly where hospital H6 is inefficient . By the way, the objective function value of 0.9615 for hospital H6's LP implies that the more efficient composite hospital produces its superior outputs by using at most 96.15% as much of each input.

Averaged output vector [Hospital H7]

$$0.769231\begin{bmatrix}19.89\\39.78\\6.63\end{bmatrix}=\begin{bmatrix}15.299\\30.59\\5.09\end{bmatrix}$$

Averaged input vector [Hospital H7]

$$0.769231\begin{bmatrix}250\\42.436\end{bmatrix}=\begin{bmatrix}192.31\\32.64\end{bmatrix}$$

Suppose we create a composite hospital by combining 0.769231 of hospital H1 with 0 of hospitals (H2,H3,H4,H5,H6,H7). The averaged output vector tells us that the composite hospital produces 15.299-15.3= -0.001 less of output 1, 30.59-30.6= -0.01 less of output 2 and 5.09-5.1= -0.01 less output 3.

From the averaged input vector for the composition hospital, we find that the composition hospital uses 192.31-200= -7.69 less input 1, 32.64-40.376= -7.736 less input 2. We know see exactly where hospital H7 is inefficient . By the way, the objective function value of 0.9609 for hospital H7's LP implies that the more efficient composite hospital produces its superior outputs by using at most 96.06% as much of each input.

The efficiency scores of each of the seven hospitals, as determined by the CCR model of Data Envelopment Analysis (DEA) using Python, are shown in Figure X. The graph unequivocally demonstrates that Hospital H1 is functioning efficiently and is situated on the efficiency frontier, having attained an efficiency value of 1. The efficiency scores of the remaining hospitals (H2–H7) are marginally below 1, indicating relative inefficiency and room for improvement in terms of output maximization or resource use. The dataset's efficient and inefficient hospitals can be quickly identified with the aid of this graphical comparison.

https://doi.org/10.71058/jodac.v9i11002

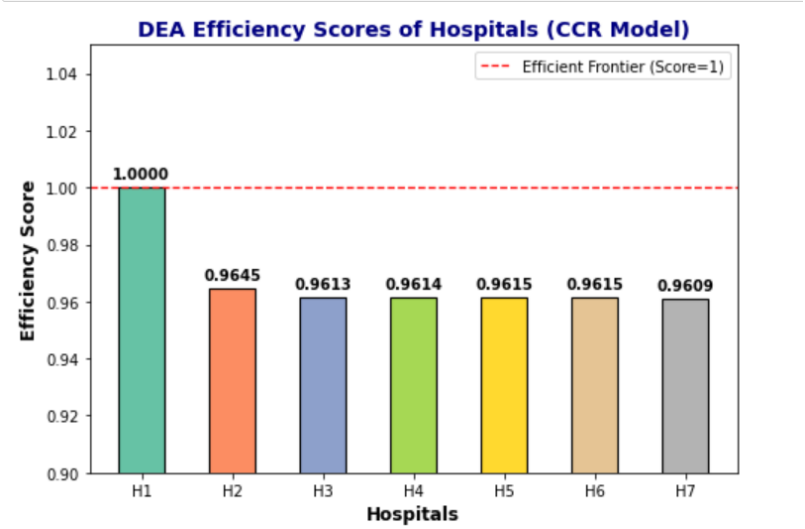

FIGURE:X

7. Conclusion

The study aimed to assess the performance of hospitals in Rajasthan. The DEA approach was applied to a sample of branches operating in seven hospitals in 2025. The variables used in the analysis were labor hours and capital, which represent the hospitals' primary inputs and patient days during month for patients under 14, between 14 and 65 and above 65, which represent the hospital' primary outputs. The study demonstrates that the typical LP strategy cannot provide as much information as the Python-based DEA strategies. The trials demonstrate that, when compared to other approaches, this method can yield intriguing results and be a suitable tool for DEA issue analysis with ambiguous data. This strategy has the benefits of visualizing, adaptability, and modesty. Hospital management found the results of the organization's hospital comparison to be beneficial.

The importance of this research lies in its contribution to both policy-making and academic understanding by identifying operational benchmarks, highlighting inefficiencies, and offering insights into resource utilization in healthcare. Given the limited existing literature on DEA applications specific to hospitals and the innovative use of PYTHON for modeling,

Author contributions: Saloni: Conceptualization, Writing of original draft, Methodology, Formal analysis, Data curation, Validation and writing–original draft preparation. Mohammad Rizwanullah (M.R.): Formal analysis, software application, Writing–review and editing, Supervision, Project administration.

Funding Open access funding provided by Manipal University Jaipur. No funding is available for present research.

Data availability All the data utilized for analysis is available in manuscript.

Declarations:

https://doi.org/10.71058/jodac.v9i11002

Ethics approval and consent to participate: No human/animal was used in the study and not based on any clinical trial

Consent for publication: No consent is required.

Competing interests: The authors declare no competing interests.

Clinical trial number: not applicable**.**

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