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Abstract: *A nonlinear energy harvesting system is proposed, consisting of a tip-loaded slender vertical cantilever beam with a strain gauge attached. The beam is oriented with its free end pointing upward and its fixed end at the bottom, and is subjected to vertical base excitations. Under such excitation, the slender beam experiences a compressive force resulting from both the base motion and the tip mass. The theory of buckling is explored, and it is shown that beyond a critical value of the crippling force, the beam undergoes buckling. To understand the system's stability, the potential energy equation is analyzed, revealing conditions for monostability, biostability, and stochastic responses. Free vibration tests and corresponding simulations show good agreement in the natural frequencies of the proposed beam. Experimental results under frequency-swept vertical excitation demonstrate that the combined inertial forces from the tip mass and base motion can transform the system's behavior from monostable to bistable. This resulted in non-linear large amplitude horizontal vibrations of the harvester. The strain gauge measurements confirm the feasibility of using this vertical cantilever beam for the efficient vibration energy harvesting.*

Keywords: *Nonlinear, Energy Harvesting, Piezoelectricity, Vibration*

1. Introduction

Due to limitations in storage facilities and availability of electric power in off-grid regions, there is a need to generate electricity from locally available energy sources and this is called energy harvesting. Energy harvesting is primarily intended for powering devices with low power consumption. Single-degree-of-freedom (SDOF) vibration energy harvesters operate most efficiently at their resonance frequency but suffer a steep decline in performance under off-resonance conditions. To address this limitation, several approaches have been investigated, including multifrequency [1] and multimode designs [2], coupled harvesters [3], hybrid transduction mechanisms [4], dynamic magnifiers [5], and nonlinear configurations [6]. Among these, nonlinear bistable systems have attracted particular interest due to their ability to broaden the operational bandwidth and improve energy capture under broadband or variable-frequency excitations. Such nonlinear systems often rely on mechanisms like magnetic interactions, bistability, and mechanical stoppers to achieve wider frequency response and higher conversion efficiency than conventional linear designs. Erturk et al. [7] demonstrated the superiority of the piezomagnetoelastic structure as a broadband energy harvester compared to the conventional configuration without magnetic buckling. Ali et al. [8] analyzed piezomagnetoelastic energy harvesters subjected to broadband random ambient excitations. A closed-form approximate expression for the ensemble average of the harvested power was derived and validated against numerical simulation results. Lan et al. [9] verified the buckling and bistable motion of a vertical slender cantilever beam under varying excitation accelerations. At a lower excitation acceleration (0.5g), the beam vibrated vertically with stability. At a higher

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acceleration (1g) the beam buckled. Two frequency components were observed and in lower modes vibration amplitudes were significantly higher. With a further increase in excitation acceleration (1.5g), the beam remained buckled, but the vibrations exhibited chaotic behavior. This phenomenon significantly increased the energy harvesting efficiency. The same specimen was studied by Lan et al. [10] to evaluate its effectiveness in statically stable, dynamically unstable, and buckled states under varying tip mass conditions. A distinct phenomenon of band of frequencies was observed at excitation frequency 14 Hz and concurrently beam vibrated at its first mode with higher amplitudes. This was due to transfer of energy of vertical high frequency vibration into horizontal low frequency large-amplitude vibration. In the existing literature, vertical cantilever beam energy harvesters have been analyzed primarily by varying parameters such as tip mass and base excitation amplitude at fixed excitation frequencies. However, there appears to be a lack of studies focusing on the system's response under frequency sweep excitation and the associated dynamic phenomena.

This letter presents a dynamic bistable piezoelectric energy harvester, comprising a vertical cantilever beam with a tip mass subjected to vertical base excitation. Experimental results demonstrate that the inertial force of the tip mass can destabilize the beam in the vertical direction, leading to buckling over time. This buckling induces a transition in the beam's stability, resulting in periodic or chaotic oscillations in the horizontal direction representing large-amplitude motion between two potential wells. Under vertical excitation, the time-varying inertial force of the tip mass transforms the system from monostable to bistable. This dynamic bistability enables the beam to oscillate with large horizontal amplitudes. By attaching piezoelectric patches to the beam, the system effectively converts high-frequency vertical excitation into horizontal vibrations, thereby enabling efficient energy harvesting. Furthermore, this configuration is capable of harvesting energy not only from low-frequency excitations but also from relatively high-frequency inputs.

The remainder of this article is organized as follows. Section 2 presents the numerical simulations in to describe the configuration of the vertical cantilever beam for the given range of frequencies. Section 3 describes the experimental setup and test rig developed to validate the proposed concept. Section 4 reports and analyzes the experimental results, highlighting the key dynamic behaviors and energy harvesting performance. Finally, conclusions summarize the main findings and outlines potential directions for future work.

2. Simulations in COMSOL Multiphysics

Initially eigenfrequency analysis is carried out in COMSOL Multiphysics software to estimate the dimensions of the harvester substructure beam. The following specifications are considered for this analysis. Active dimensions of MFC: 28×7×0.3 mm, overall dimensions of MFC: 37×10×0.3 mm. The simulations are carried out by bonding the active dimensions of the MFC with the substructure, and with a tip mass. The material properties for this study are taken from standard charts and catalogues and are as follows: - Copper: Young's modulus 110 GPa, density 8906 kg/m^3 and Poisson's ratio 0.3; MFC: Young's modulus 15.857 GPa and density 5440 kg/m^3.

Figure 1(a) shows the discretized finite element model of the vertical cantilever harvester beam in COMSOL Multiphysics 3a software. The beam is discretized with in-built tetrahedron elements. The eigenfrequency analysis is performed and the mode shapes are obtained in the frequency range considered for the analysis. The first mode frequency is found to be at 1.871 Hz, second and third modes are at 15.67 Hz and 47.546 Hz, Figure 1(b), (c) and (d) shows their mode shapes respectively.

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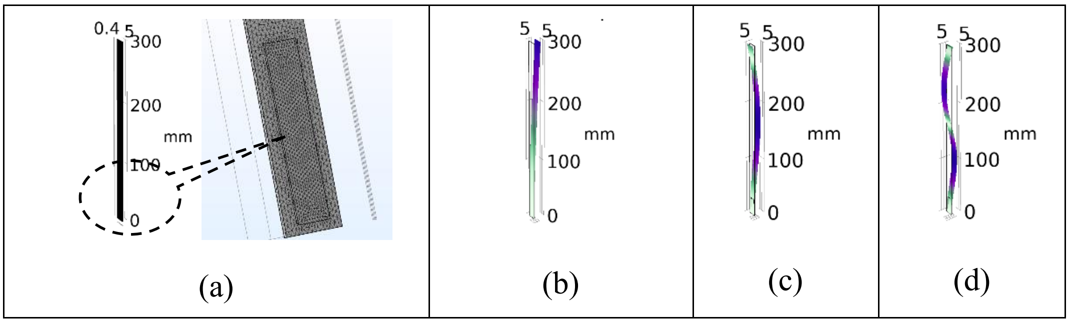

Figure 1. Vertical cantilever harvester beam with tip mass 8.48 gm (a) Finite Element Model (b) Mode-1 (c) Mode-2 (d) Mode-3.

3. Experimental setup

To comprehensively evaluate the system's performance and validate the energy harvesting mechanism, a series of corresponding experiments were designed and conducted. The experimental setup consists of a vertical cantilever harvester, electrodynamic vibration shaker with an amplifier used for the base excitation, signal generator used to input the required signal form, data acquisition system used to measure and store input and output data. A miniature accelerometer used to measure the base excitation acceleration. The output voltage from the cantilever beam piezoelectric harvester depends on the strain induced near the fixed end of the beam. To understand the feasibility of the vertical beam as a harvester, initially beam attached with strain gauge instead of piezoelectric patch near the fixed end. During the steady state forced vibration experiment, the frequency is swept from 5 Hz to 60 Hz. The measured strain readings are from zero to peak. Commercial DewesoftX software is used to read the data from the data acquisition system. Figure 2 shows the complete experimental setup used to perform the parametric studies.

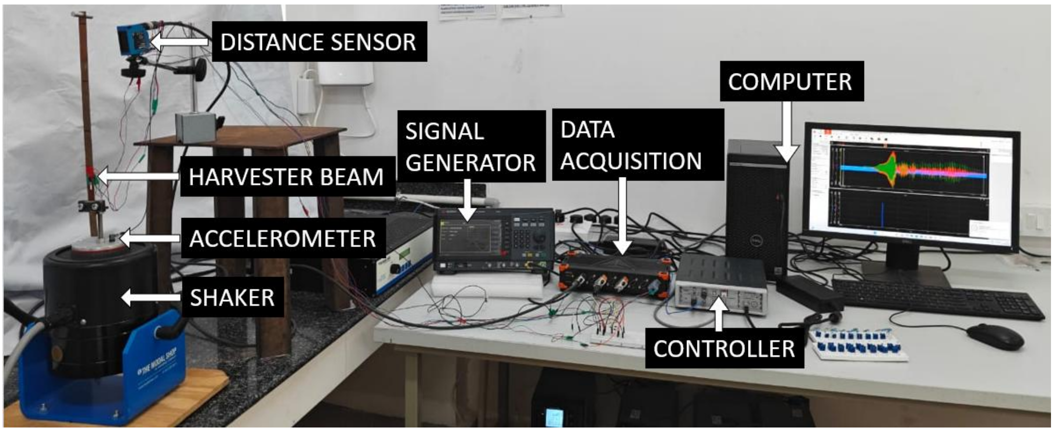

Figure 2. A comprehensive experimental setup

4. Results & Discussions

4.1 Free vibration analysis

The free vibration analysis is performed with a tip mass of 8.48 gm. The beam is mounted on a fixed table in vertical direction and lateral disturbances are given. The following Figure 3(a) shows the plot of peak to peak output voltage amplitude from strain Vs time. It is observed that at the beginning multiple modes are appeared, later followed the first mode vibrations with gradual decay in amplitude. Figure 3(b) shows the frequency response plot. The measured first mode frequency is found to be at 1.83 Hz and 16.85 Hz and 50.17 Hz for second and third modes respectively. Whereas predicted first three mode frequencies are 1.87 Hz, 15.7 Hz and 47.54 Hz.

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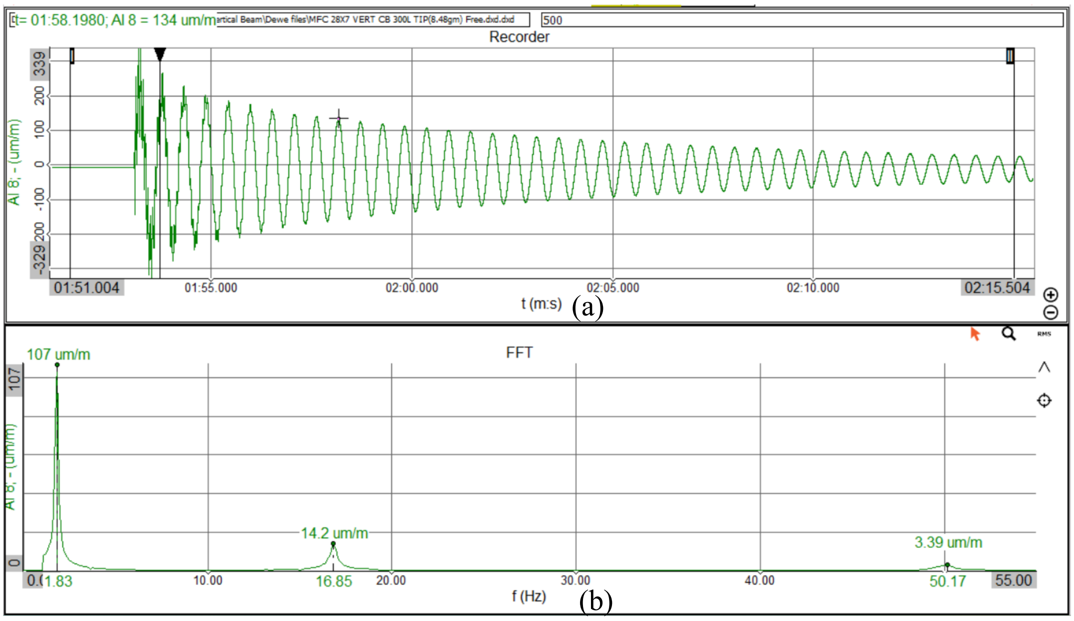

Figure 3. Free vibration analysis vertical cantilever harvester beam without tip mass (a) Voltage Vs time (b) Frequency response

It is concluded from the free vibrations tests that the predicted and measured out of plane natural modes of vertical cantilever harvester beam with a tip mass are in good agreement. Hence this beam is used to perform the further parametric studies.

4.2 Forced/steady state vibration test

When a slender beam excited by vertical excitation, the inertia force of tip mass becomes time-variant which transforms the beam from monostable to the bistable. The dynamic bistable property is the cause for the vertical beam to vibrate at a large displacement in horizontal direction. If f_{cr} is the maximum axial compressive force in vertical cantilever beam. According to the theory of beam stability, the beam will be in buckling, if the $f_{cr,}$ is larger than the axial force. The beam will be in vertically stable position if the gravity force is less than the critical force. If beam is excited by acceleration $a(t)$ in vertical direction, the inertia force produced by the tip mass is $m_0(g+a(t))$, due to this the axial force of the beam will change with base excitation. If the tip mass and excitation acceleration are designed such that the resulting dynamic axial force exceeds the beam's critical load, the beam will gradually lose stability and undergo buckling over time. It found from the given geometric and material properties of the harvester beam, the crippling force is found to be 15.64 gm. Hence with the available tip masses, vertical cantilever harvester beam is measured for its steady state performance for the tip mass 15.90 gm.

The vertical beam cantilever harvester is mounted on a shaker table with vertical cantilever fixtures. The input base excitation is given by shaker through signal generator and amplifier. The performance of this harvester is studied for the output open circuit voltage from the MFC. The performance studied by sweeping the frequency between 5 Hz and 60 Hz. The minimum excitation frequency of 5 Hz is chosen due to the limitation of the shaker.

The plots of input base excitation acceleration, output voltage Vs time and plots of input excitation acceleration, output voltage Vs frequency are taken from the Dewesoft for the postprocessing analysis. The black color cursor is placed at the locations of maximum amplitudes in time plots to find values of the different resonance mode frequencies in frequency response plots. The present experimental setup with base excitation is given by shaker through signal generator and amplifier, increased the excitation acceleration gradually till 45 Hz, thereafter remained constant as shown in Figure 4(a). The cursor is placed at the first maximum value in amplitude vs time plot can be seen in Figure 4(b) and

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(b). The measured natural frequency of second mode is found to be 15.75 Hz. This is shown in frequency response plots, Figures 4(c) and (d). In Figure 4(a), the excitation acceleration is found to be 0.8 g. The maximum output strain at this frequency is found to be 190 µm/m. The input base excitation frequency and output voltage frequency are found to be same and is 15. 75 Hz. This indicates that vertical cantilever beam vibrating at its lateral bending mode and the vibrations are linear.

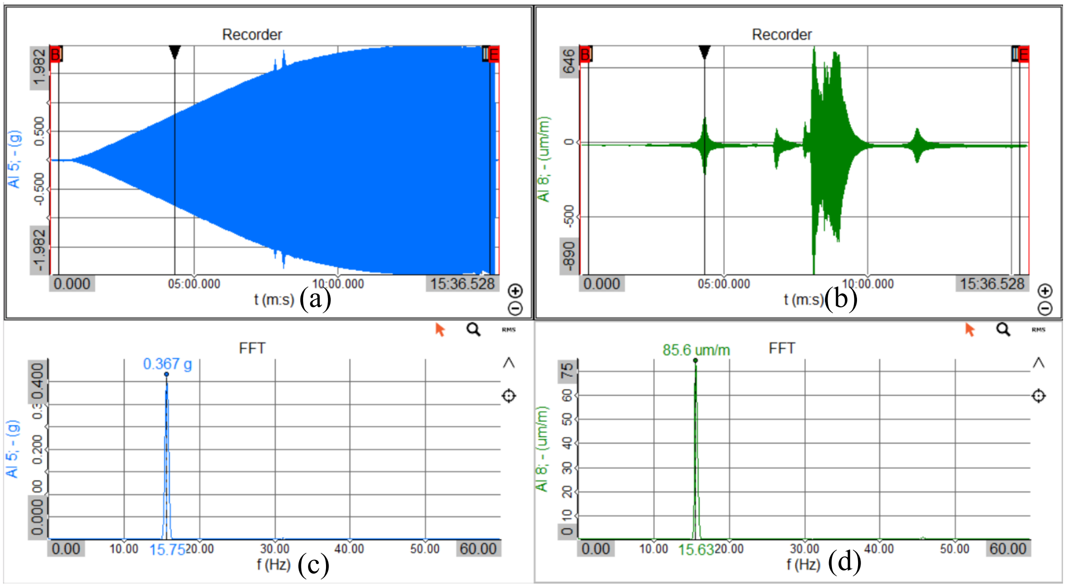

Figure 4. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the second mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

Further the cursor is moved to the next peak value in time plot. In Figure 5(a), the excitation acceleration amplitude found to be 1.4 g and excitation frequency 25.70 Hz in Figure 5(b) respectively. The time response voltage Figure 5(b) shown a peak at this frequency. The voltage frequency response plots Figure 5(c) and (d) have shown two peaks at 1.22 Hz and 45.10 Hz. These two peaks are the first and third natural modes of the beam. A side band of frequencies are observed at these two peaks. These side bands are predominant at the third mode than at the first mode. This shown that the harvester beam is vibrating at multiple natural modes of frequencies that is at first and third mode at this excitation frequency and first mode is observed to be more dominant than the third mode. Figure 6(a) and (b) shows the enlarged views of time plots of this coupled natural modes phenomenon. The vertical vibration energy of high frequency is transferred into low frequency high amplitude and high frequency low amplitude vibrations. Besides this there appears many other peaks around third natural mode 45.10 Hz. This phenomenon is a manifestation of side band. Figure 7(a) and (b) shows enlarged views of this phenomenon at third natural mode of the beam. It should be noted that the difference between every neighboring peak is 0.6 Hz, half the frequency of the first mode. This sideband is originated from the nonlinearity and the modulation of the first mode on the synchro response. The nonlinear internal resonance results in the appearance of the first mode in response. The first mode and its harmonic components modulate the synchro response and cause the sideband in the spectrum of response. The maximum strain is found to be 130 µm/m.

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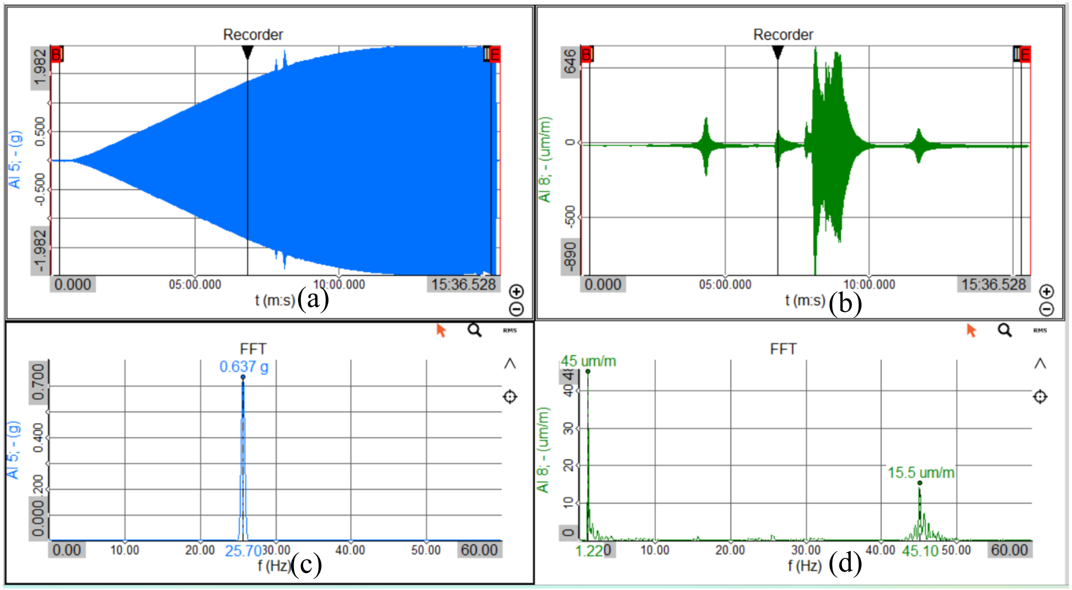

Figure 5. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the sideband mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

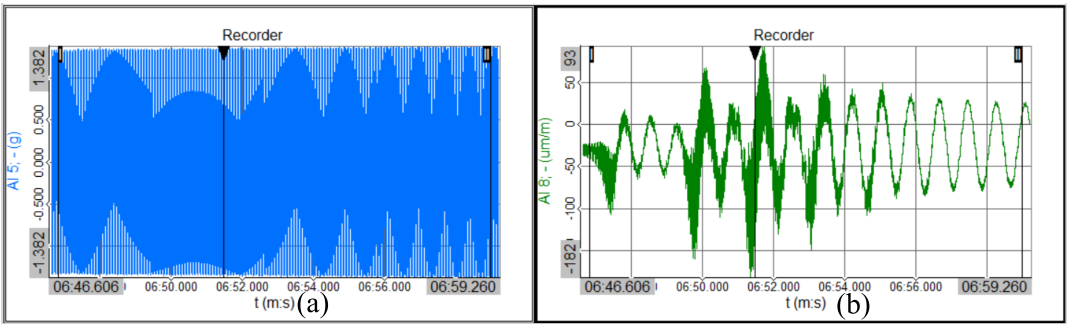

Figure 6. Enlarged view of forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the sideband mode (a) Base excitation acceleration Vs time (b) Voltage Vs time.

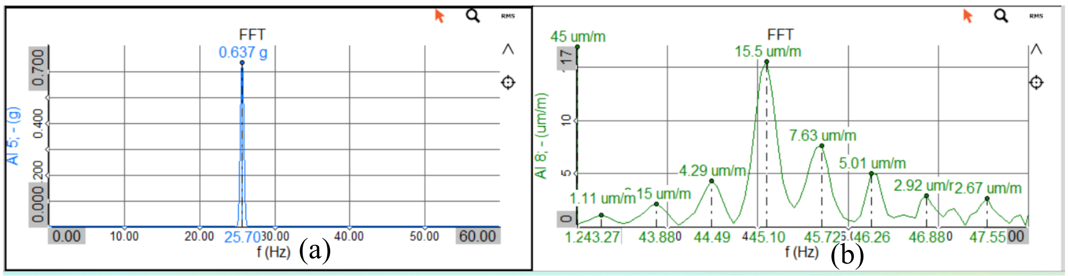

Figure 7. Enlarged view of forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the sideband mode (a) Base excitation acceleration Vs time (b) Voltage Vs time.

In between the base excitation frequency sweep from 29 Hz to 38 Hz a special phenomenon is observed. At an excitation acceleration sweep of 29.91 Hz at 1.54 g, the beam started vibrating at multiple frequencies viz 1.83 Hz, 3.72 Hz and at excitation frequency 29.91 Hz. Figure 8(a) and (b) shows the excitation acceleration and output voltage plots in time domain respectively. Figure 8(c) and (d) shows the frequency response plots.

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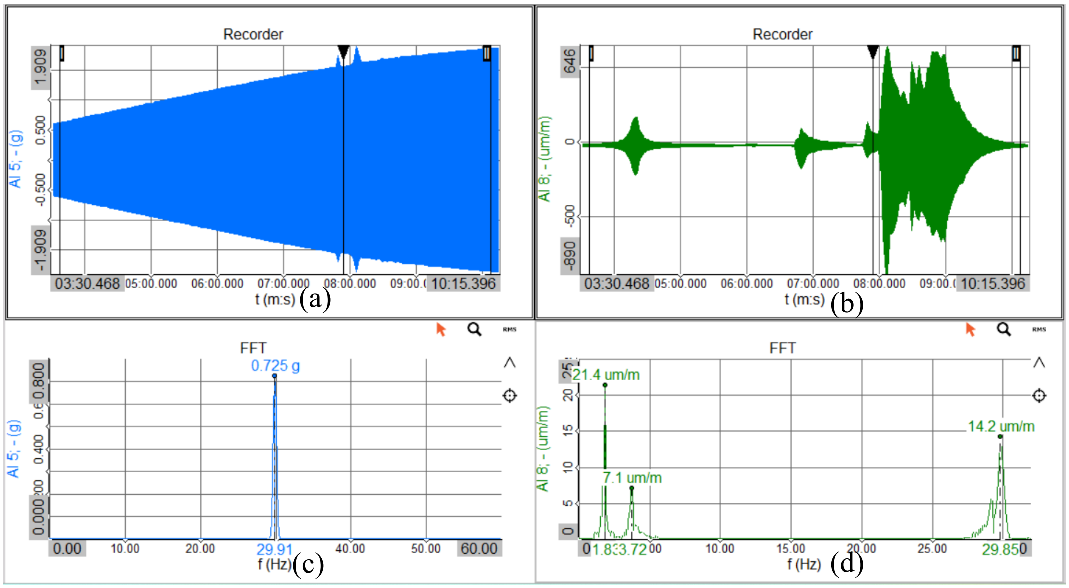

Figure 8. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the coupled mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

Further gradual increase in the frequency, a coupled integral mode phenomenon is observed at an excitation frequency of 30.7 Hz shown in Figure 9(c). The beam initially started vibrating at coupled first, second and third lateral bending modes at frequencies 1.22 Hz, 15.68 Hz and 45 Hz respectively and achieved a maximum amplitude. It is observed that the maximum output strain is found to be 760 µm/m shown in Figure 9(b). The excitation high frequency vibration energy is transferred to the low frequencies at first and second modes of the beam with high amplitudes. Here the amplitude of second mode is found to be more dominant than that of first mode.

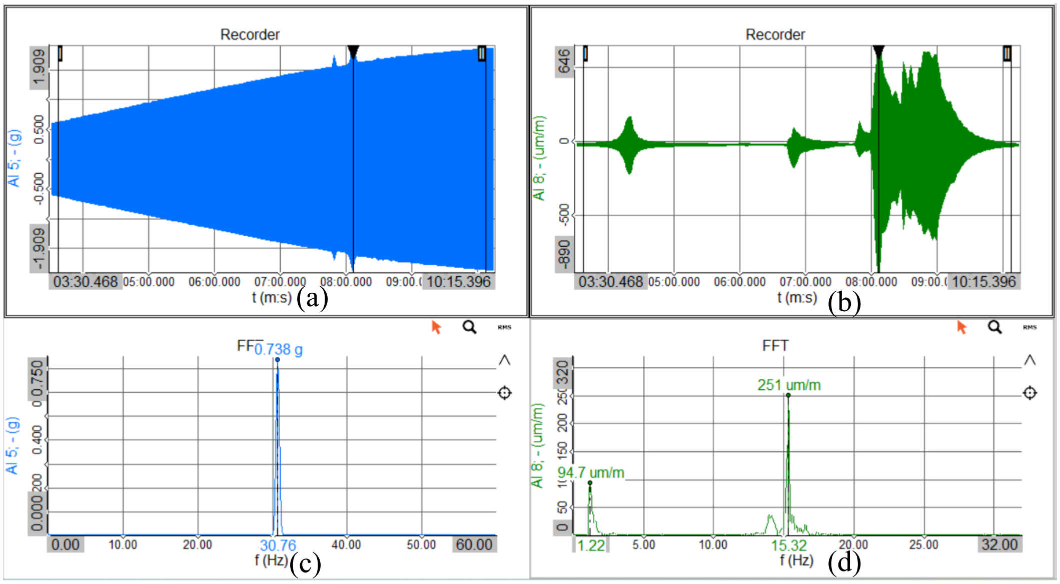

Figure 9. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the coupled mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

Further gradual increase in excitation frequency the amplitude of second mode decreased gradually and in the meanwhile first mode started gaining higher amplitude, till the excitation frequency 36 Hz. The energy of second mode completely diminished, further gradual increase in excitation frequency, first mode amplitude reduced gradually till 39 Hz. This indicates there exists a strong coupling between the first and second modes and the

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excitation vibration energy is transferred to these modes. This phenomenon can be clearly seen in frequency response Figures 10(a), (b), (c) and (d) and 11(a), (b), (c) and (d). Figure 12(a) and (b) shows the enlarged views of base excitation and time response plots.

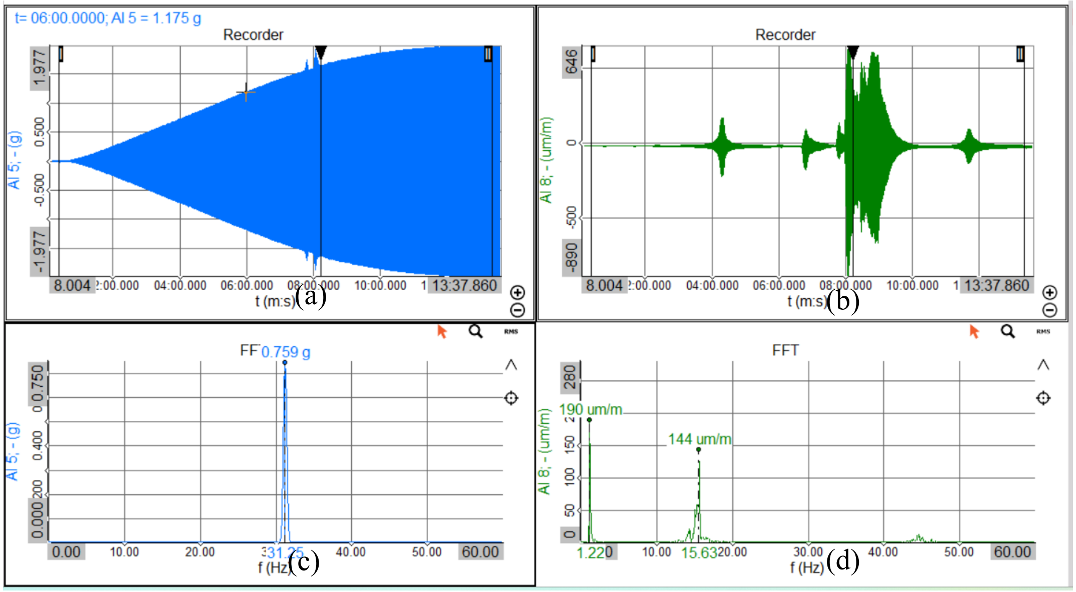

Figure 10. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the coupled mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

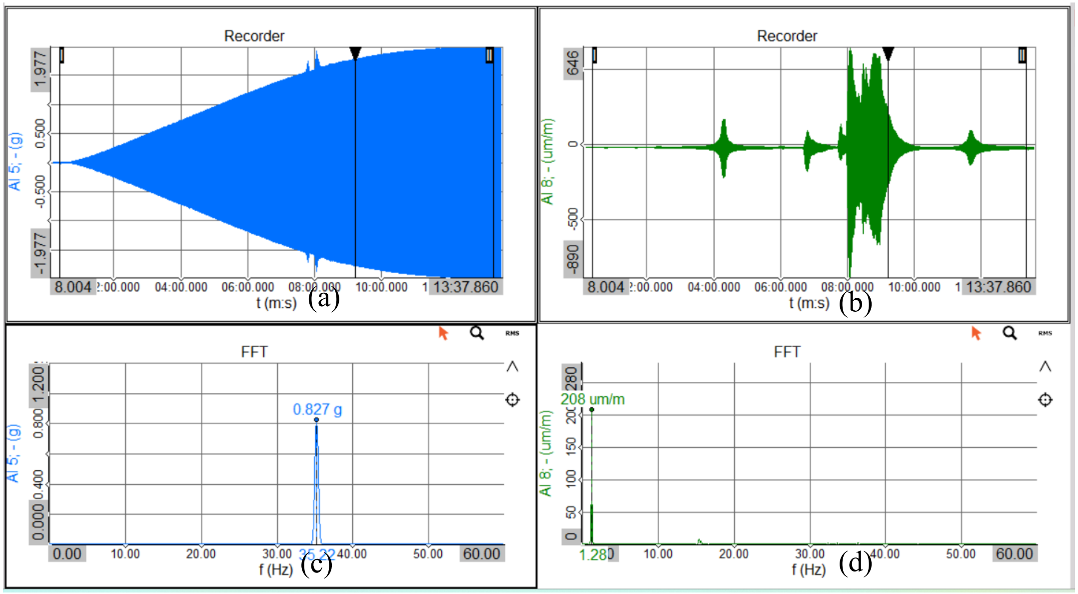

Figure 11. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the coupled mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

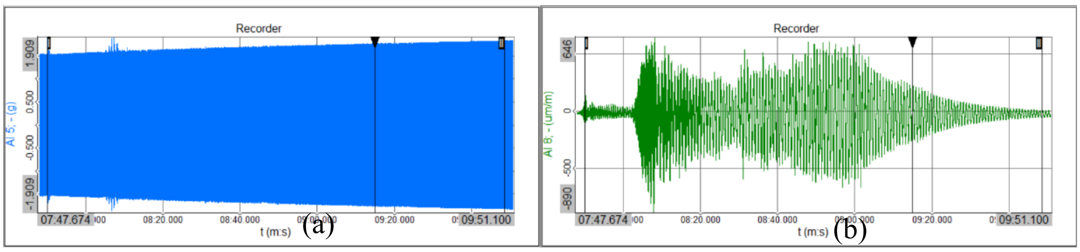

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Figure 12. Zoomed in view of Forced vibration analysis vertical cantilever harvester beam with a tip mass of 15.90 gm for the coupled modes (a) Base excitation acceleration Vs time (b) Voltage Vs time.

Further the cursor is moved to the next peak value in time plot. The next natural third mode is appeared at an excitation frequency of 45.1 Hz and the frequency of output voltage remained as that of excitation. In Figure 13(a), the excitation acceleration is found to be 2.0 g. The maximum strain at this frequency is found to be 110 μm/m shown in Figure 14(b). It is found that both excitation acceleration and output voltage frequencies are same and is 45.1 Hz. This means that the beam is vibrating at its third lateral bending mode for the given vertical base excitation acceleration showing linear behaviour.

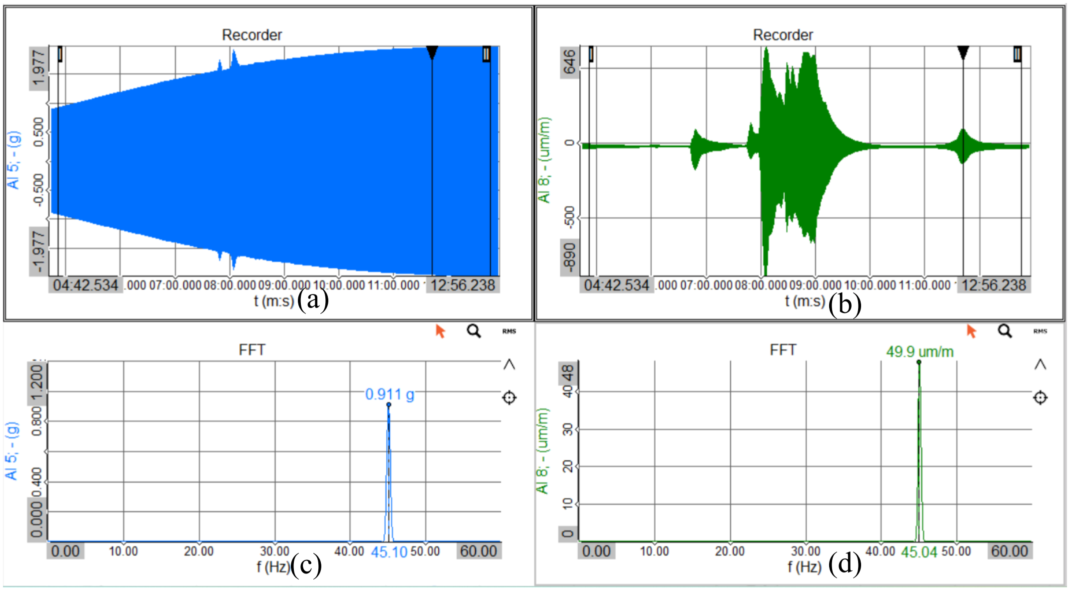

Figure 13. Forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the third mode (a) Base excitation acceleration Vs time (b) Voltage Vs time (c) Base excitation acceleration frequency response (d) Voltage frequency response.

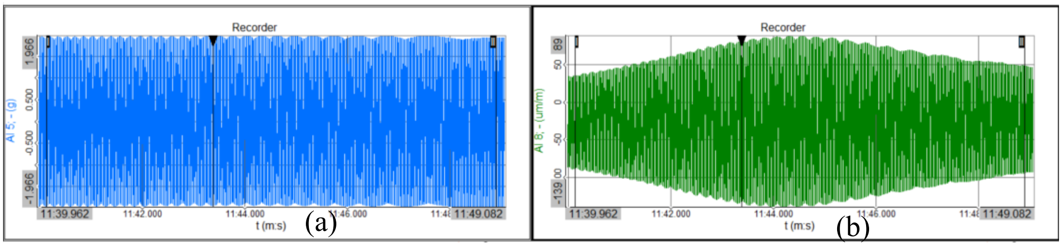

Figure 14. Enlarged forced vibration analysis of vertical cantilever harvester beam with a tip mass of 15.90 gm for the third mode (a) Base excitation acceleration Vs time (b) Voltage Vs time.

Conclusions

The proposed vertical beam harvester, designed for operation under vertical base excitations, was assessed for feasibility by mounting a strain gauge near the fixed end to measure strain-induced output. The tip mass was found to play a critical role in stability, with the beam transitioning from monostable to bistable behavior above the crippling load. Free vibration experiments showed strong agreement with numerical simulations. Nonlinear effects and first-mode modulation of the synchronous response produced sidebands in the response spectrum at a base excitation of 25.7 Hz and 1.4 g. Additionally, mode coupling between the first and second modes was observed at 30.7 Hz and 1.6 g, generating strain outputs from 230 μm/m to 760 μm/m and persisting up to 34 Hz. This strain range confirms the harvester's potential for vibration energy harvesting when

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integrated with piezoelectric materials, while the mode coupling phenomenon enables efficient transfer of high-frequency vibration energy into lower-frequency modes with high amplitudes, further enhancing harvesting efficiency.

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