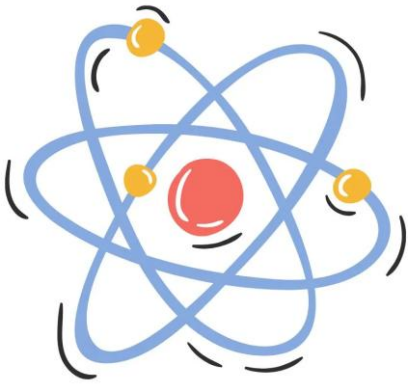

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Abstract: *Modern Communication systems require network security in order to provide safe data transfer, efficient routing and fault tolerance measures. Total Prime Graceful labeling facilitates the identification of security threats within the network infrastructure. Total Prime Graceful labeling is an intriguing concept in graph theory that has applications in many different domains because of its special qualities and possible uses. The practical uses of total prime graceful labeling in diverse domains of network security contexts are examined in this study.*

Keywords: Total Prime Graceful Labeling, Star Graph, SIEM, VPN, Access Control Systems

1. Introduction

The finite simple undirected graphs G(V, E) are considered in this work. The graph G is composed of the set of vertices V(G), set of elements E(G) and its incidence relation. Rosa [1] was the first to introduce graph labeling, who also provided several graph labeling methods. She specifically came up with the phrase $\beta-$ labeling and it was renamed by Solomon [12] as graceful labeling. Roger Entringer initiated prime labeling. Later, Tout et al. [2] examined several graph types to determine which permit prime labeling. Total prime graph was presented by Ravi et al.[6] along with the outcomes for a few graphs that allow total prime labeling. The survey conducted by Gallian [4] on various graph labeling techniques has been referred. Super total graceful graphs were introduced by Subbiah et al. [11], who also discovered certain graphs that allow for super total graceful labeling. Selvarajan and Subramoniam [13] introduced a new labeling method called Prime Graceful Labeling in 2018. Later, Sayan Panma and Penying Rochanakul [8] created prime-graceful numbers and k-prime-graceful labeling. Nandhini S.P. and Pooja Lakshmi B. [10] not only extended the prime graceful labeling to specific graphs but also expanded on the cardinality of edges in the triangular snake graph. In 2024, the total prime graceful labeling technique was introduced by Keerthika and Kowsalya [5] and the new technique is analyzed for some simple graphs. In 2014, Prasanna et al. [7] have examined how labeling techniques enhance channel assignment in communication networks with applications spanning network security, addressing protocols and social network analysis. Subsequently, Fridah [3] review has analyzed various software security methodologies and frameworks designed to mitigate vulnerabilities. The findings highlighted that exploitable code weaknesses have enabled attackers to compromise computerized systems representing a significant threat to contemporary organizational networks in 2023. The cybersecurity sector has consequently developed multiple approaches to strengthen software reliability. In 2024, Lyu et al. [9] have introduced a cloud-based control switching framework employing multiple controllers for improved stability in unpredictable network conditions. Recently, Zhao et al. [14] proposed a cross-domain federated graph learning method for identifying network traffic anomalies to counter evolving cybersecurity challenges in

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2025. Graph models and labeling techniques have become essential tools for addressing complex challenges in modern network systems and mathematical frameworks provide systematic approaches to optimize communication protocols, detect anomalies, and manage interconnected infrastructures in increasingly dynamic environments. These development and application remain critical for advancing cybersecurity, network performance and the resilience of contemporary digital systems. In this paper, we examine the idea of security software utilizing total prime graceful labeling and shown the structured representation of the network structure of the software which enables centralized management, monitoring, identifying security threats prompting further investigation and remediation actions.

2. Total Prime Graceful labeling of Star graph in Network Security

A graph G with p vertices and q edges admits **total prime graceful labeling** if the bijective function $\omega: V(G) \cup E(G) \rightarrow \{1,2,3,..,p+q\}$ such that $\omega(uv) = |\omega(u) - \omega(v)|$, $\forall\ uv \in E(G)$ and GCD of $(\omega(u), \omega(v), \omega(uv)) = 1$. (i.e.), the labels assigned to u and v and the incident edge *uv* are relatively prime. The graph which admits total prime graceful labeling is called total prime graceful graph [5].

A **Star graph** ($K_{1,n}$) [5] is a tree with n leaves with degree 1 and 1 internal node with degree n.

Theorem:[5]
For any positive integers n, the star graph $K_{1,n}$ admits total prime graceful labeling.
Proof:
Let V ($K_{1,n}$) = {u} ∪ {v_i : $1 \le i \le n$} and E($K_{1,n}$) = {uv_i : $1 \le i \le n$}. Define the vertex and edge labeling as, ω: V (G) ∪ E(G) → {1,2,3,....,2*n*+1}. Label the vertex of degree n with 1. (i.e), $\omega(u) = 1$. The n-pendant vertices are designated as $\omega(v_i) = 2i + 1$, ∀ $1 \le i \le n$. The edge set are designated as E(G)=2i, ∀ $1 \le i \le n$. It is determined by $\omega(uv_i) = |\omega(u) - \omega(v_i)| = |1 - (2i + 1)| = 2i$. The adjacent vertices and incident edges are relatively prime. (i.e.), GCD $(\omega(u), \omega(v_i), \omega(uv_i))$ = GCD $(1, 2i + 1, 2i) = 1$ Thus, ω admits total prime graceful labeling. Therefore, $K_{1,n}$ is a total prime graceful graph.

Total prime graceful labeling can be applied to graphs representing network security monitoring or network monitoring infrastructure analysis. The unique labels can facilitate secure communication between components and the distinct differences can aid in event analysis. The star graph topology can be used to model several network in the context of total prime graceful labeling including security monitoring settings, access control systems and centralized network infrastructures.

The total prime graceful labeling of star graph model involves the following steps:
1) Network devices, sensors or endpoints that communicate with the central node to enforce security rules, identify risks, or to enable secure communication are represented by pendant nodes which connect with the central point of control or monitoring.
2) Determine the total of number of vertices and edges in the star graph.
3) Represent the network or system as a star graph, where the central vertex represents the main component (e.g., a central server, security device etc.) and the other vertices represent additional components or segments connected to the central node. 4) Assign a label of 1 to the central vertex of the star graph.
4) The n-pendant vertices are designated as 2n+1 for $1 \le i \le n$.
5) For each edge connecting the central vertex (vertex u with label 1) to another vertex, the absolute difference between the labels of u and v denoted as $|\omega(u) - \omega(v)|$ must be a distinct even number. This property ensures that the differences between the central vertex label and the labels of the connected vertices are also distinct even numbers which are designated as 2n for $1 \le i \le n$.

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6) Assign unique numbers as labels to the edges connecting the central vertex to each of the other vertices. These numbers serve as unique identifiers for the communication links or connections between the central node and the other nodes.
7) The total prime graceful labeling of the star graph provides an additional layer of security by incorporating odd and even numbers as labels for both vertices and edges enhancing secure communication.

3. Application

3.1 Security Information and Event Management (SIEM)

Central Node: SIEM server
Pendant Nodes: Network devices, Firewalls, and IDS devices

Total prime graceful labeling of a star graph in the context of a SIEM server:

1. Understanding Total Prime Graceful Labeling:
Total prime graceful labeling is the process of employing numbers to give distinct labels to network resources, devices and access control. These labels are organized to help with the analysis and management of security events as well as to communicate information about the entities they represent.
2. Star Graph Representation:
The relationship between the SIEM server (central node) and different log sources (pendant nodes) such as network devices, firewalls, IDS devices and other security appliances is represented by a star graph. Security event logs are produced by each pendant node which are gathered and forwarded to the SIEM server for analysis and correlation.
3. Assigning Numbers:
Every node in the star graph which includes the pendant log sources and SIEM server has a unique label. Assume that the SIEM server is allocated number 1. Sequential odd numbers ranging from 3 are assigned to each pendant log source (endpoint). A network device for instance might be identified by the number 3, a firewall by the number 5, an IDS device by the odd number 7 and so forth.
4. Example:
Consider a SIEM server setup with a star graph topology consisting of the following nodes:
SIEM Server (Central node): Responsible for collecting and analyzing security event logs.
Pendant Log Sources (Endpoints): Include Firewalls, IDS devices, Network devices and other security appliances.
1. Firewall: The firewall at the network perimeter sends security event data to the SIEM server. This includes logs of blocked traffic, intrusion attempts, and other security-related events.
2. Intrusion Detection System (IDS): An IDS deployed within the network also sends event data to the SIEM server. This includes alerts generated by the IDS for suspicious or malicious activity detected on the network.
3. Network Devices: Routers, switches, and other network devices send logs and event data to the SIEM server. This data includes information about network traffic, device status, and configuration changes.

Let's assign labels to these components:
SIEM Server (VMS): Label = 1
Network devices, Firewalls, IDS Devices: Labels = 3, 5, 7, 9, 11, ... (odd numbers)

Let's now create links or edges between these parts according to their functions and connections inside the SIEM Server. The absolute difference between the labels of u and v represented as $|f(u) - f(v)|$ for each edge joining the center vertex (vertex u with label 1) to a different vertex (vertex v) must be a different even number. This feature guarantees that the labels of the connected vertices and the central vertex label differ by unique even values which are designated as $2n$ for $1 \leq i \leq n$. The graphical representation of security information and event management is shown in Figure 1.

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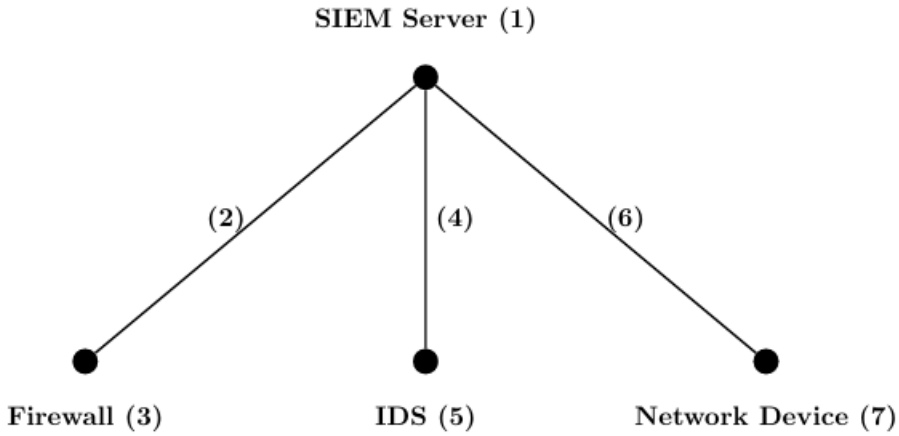

Figure 1. SIEM Server Utilizing Total Prime Graceful Labeling

A structured representation of the network topology is produced for the SIEM Server by the aid of total prime graceful labeling. Any variations or irregularities in the labeling pattern may point to possible security risks or efforts at illegal access, prompting additional research and remedial measures.

Identifying and Investigating Duplicate Labels:

1)Assume if security administrators while monitoring the network discover the duplication in number labeling or variations in the labeling pattern. Then the administrator receives an alert indicating the suspicious activity conducted towards server. To find out the reason for the duplication and any possible effects on network security they investigate further.

2)Administrators move quickly to restore the integrity of the network configuration after realizing the safety issues provided by the duplicate labels. To stop similar occurrences in the future, this may require isolating the affected devices returning them to their original settings and placing it in the place which has extra security measures.

3)To strengthen security measures and the analysis of security events, total prime graceful labeling of a star graph in a SIEM server involves providing structured unique number labeling to the SIEM server and its log sources. Within the SIEM context this labeling technique makes effective security event correlation and incident response possible.

3.2 Virtual Private Networks (VPN)

Central Node: VPN server

Pendant Nodes: VPN clients, network devices

Total prime graceful labeling of a star graph in the context of a VPN:

1. Understanding Total Prime Graceful Labeling: The process of labeling nodes and edges in a graph where each edge is uniquely labeled depending on the absolute difference of the labels of its endpoint and the label assigned to distinct node u, v and the incident edge uv are relatively prime is known as total prime graceful labeling. Based on the structured labeling methodology, Total Prime graceful labeling enables organizations to implement additional security features including access control, authentication, and encryption. By doing this the VPN network's security posture is improved and sensitive data is shielded from illegal access and interception.

2. Virtual Private Network (VPN): A Virtual Private Network (VPN) is a private, encrypted communication channel that is set up over an open network such the internet to protect data transmission between branch offices or remote users and the corporate network. VPNs

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safeguard the integrity and confidentiality of data by using authentication and encryption techniques.

3. Structured Star graph Labeling for VPN Components: An organized depiction of the VPN topology is produced by using total prime graceful labeling on VPN servers and client devices connected to the network. A distinct label is assigned to each VPN server and client device enabling centralized management, monitoring and control of VPN connections.

4. Example:

Consider a VPN network deployed within a corporate environment to provide secure remote access for employees working from home or traveling. The VPN network consists of the following components:

VPN Server: Central server responsible for managing VPN connections and providing access to corporate resources.

VPN Clients: Devices used by remote users to establish VPN connections with the VPN server and access corporate resources securely.

Let's assign labels to these components:

VPN Server: Label = 1

VPN Clients: Labels = 3, 5, 7, 9, 11, ... (odd numbers)

Now, let's establish connections (edges) between these components based on their roles and relationships within the VPN network. For example, VPN clients establish encrypted tunnels with the VPN server to transmit data securely over the internet. This property ensures that the differences between the central vertex label and the labels of the connected vertices are also distinct even numbers which are designated as $2n$ for $1 \leq i \leq n$. The graphical representation of VPN system is shown in Figure 2.

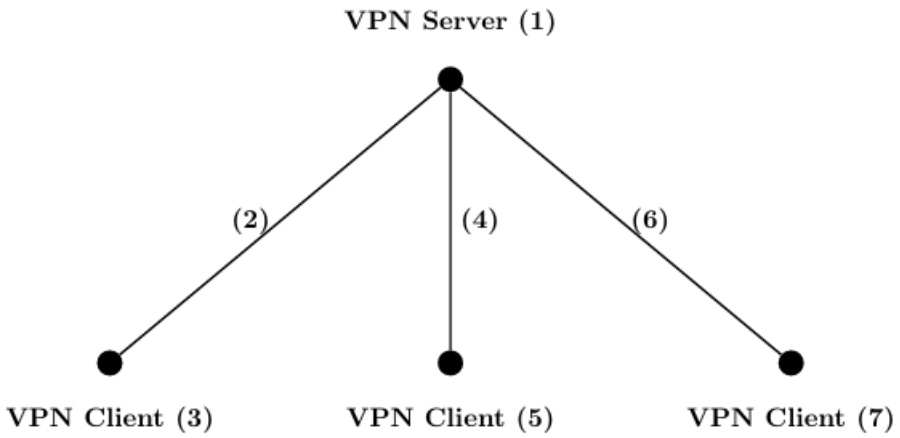

Figure 2. VPN Server Utilizing Total Prime Graceful Labeling

Identifying and Investigating Duplicate Labels:

1)Suppose a security threat occurs where an attacker gains unauthorized access to the network and manipulates the configuration to assign the same odd number label to both Gateway server and Client Device.

2)Administrators safely manage and configure VPN components using the labeled network architecture. This involves setting up VPN policies, configuring access restrictions and making certain that the VPN network is only accessible by authorized users and devices.

3)A structured representation of the network topology is produced by applying total prime graceful labeling to the VPN network which makes it possible to effectively manage, monitor, and control VPN connections. Any variations or irregularities in the labeling pattern may point to possible security risks or efforts at illegal access, necessitating additional research and corrective action.

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3.3 Access Control Systems

Central Node: Access control server

Pendant Nodes: Card readers, biometric scanners, door locks

Total prime graceful labeling of a star graph in the context of access control system:

1. Understanding Total Prime Graceful Labeling: The process of labeling nodes and edges in a graph where each edge is uniquely labeled depending on the absolute difference of the labels of its endpoint and the label assigned to distinct node u, v and the incident edge uv are relatively prime is known as total prime graceful labeling. Enterprises can apply enhanced safety features like access control policies, authentication methods and audit logs based on the structured labeling scheme with Total Prime graceful labeling. By doing this the access control system's security posture is strengthened and illegal access and security breaches are avoided.

2. Access Control Systems: Access control systems are security tools intended to govern and monitor internal organization access to digital or physical resources. Card readers, biometric scanners, door locks, access control panels and access control servers are a few examples of the equipment that make up these systems.

3.Structured Star graph Labeling for Access Control Devices: A structured representation of the access control system is produced by applying total prime graceful labeling to the resources and access control devices. To enable centralized management, monitoring and control of access rights and permissions, each device is given a unique label.

4. Example:

Consider an access control system deployed within an organization's premises. The access control system consists of the following devices:

Access Control Server (ACS): Central server responsible for managing access control policies, user authentication and access logs.

Card Readers: Devices installed at entry points to read access cards and grant or deny access based on user credentials.

Biometric Scanners: Devices used for biometric authentication, such as fingerprint scanners or facial recognition systems.

Door Locks: Mechanisms installed on doors to control physical access to rooms or areas.

Let's assign labels to these devices:

Access Control Server (ACS): Label = 1

Card Readers, Biometric Scanners, Door Locks: Labels = 3, 5, 7, 9,11, ... (odd numbers)

Now, let's establish links or edges between these devices according to their purposes and connections inside the access control system. For instance, all other devices can communicate with the Access Control Server (ACS) while Card Readers and Biometric Scanners may connect to the Access Control Server for authorization and authentication purposes. This feature guarantees that the labels of the connected vertices and the center vertex label differ by separate even numbers denoted as $2n$ for $1 \leq i \leq n$. The graphical representation of access control system is shown in Figure 3.

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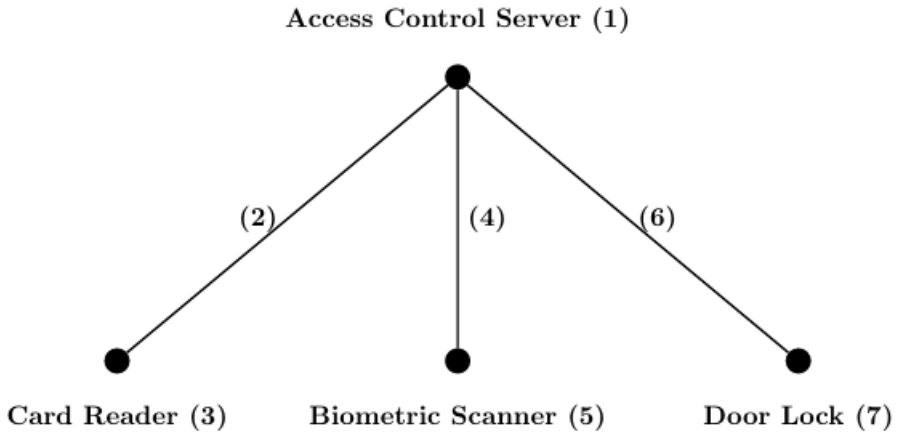

Figure 3. Access Control Server Utilizing Total Prime Graceful Labeling

Identifying and Investigating Duplicate Labels:

1)Due to label duplication, the central authentication server which expects unique numbers for every device may interpret events and access control policies received from the server and access point incorrectly. This may result in inaccurate access control decisions and even unapproved network access. Confidentiality may be lost if unauthorized access obtains information that they shouldn't have leading to loss of confidentiality. This can damage trust with stakeholders, partners, and customers may suffer as a consequence. Unauthorized users may alter or corrupt data in addition to seeing it. This might result in a spread of false information or the manipulation of important data.

2)Administrators use the labeled access control components to configure and manage access control policies effectively. This includes defining access control rules, assigning permissions and ensuring that access control configurations align with organizational security requirements.

3)A structured representation of the system's topology is generated by applying total prime graceful labeling to the access control system. This enables centralized management, monitoring and control of access rights and permissions. Any variations or irregularities in the labeling pattern may point to possible security risks or efforts at illegal access which suggesting additional research and corrective action.

4 Conclusion

As networks continue to grow in complexity and the need for security measures increases, the total prime graceful labeling has the potential to implement secure communication systems. Furthermore, total prime graceful labeling has proven secure communication in network security. Total prime graceful labeling is therefore important for applications beyond theoretical graph theory and is a useful tool for tackling real-world problems in a various domain.

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