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FRACTIONAL ORDER METHODS FOR EDGE DETECTION: A REVIEW

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Abstract: *Edge detection is an essential task in image processing and computer vision. In this context, fractional computation has emerged as an innovative technique capable of addressing the limitations of traditional methods. This article systematically reviews the application of fractional calculus in edge detection, evaluating theoretical and practical approaches, as well as methodologies for determining fractional order. Recent advances, such as fractional operators that improve edge and detail accuracy in complex images, are highlighted. However, challenges remain, such as the standardization of fractional order and its adaptation to various contexts. The study reflects on current strengths and limitations, suggesting future directions for research.*

Keywords: Edge Detection, Image Processing, Fractional Methods, Parameter Optimization

1. Introduction

Fractional calculus has emerged as a powerful tool in the analysis and control of dynamic systems, providing greater flexibility in the modeling of complex phenomena.

In the context of edge detection, traditional methods based on edge detection operators, such as Sobel, Roberts and Canny filters, have been widely used due to their effectiveness and simplicity. However, these methods are limited by their ability to capture fine details and complex variations in images. The introduction of fractional operators in edge detection has overcome some of these limitations, providing an improved ability to capture edges with greater accuracy and adaptability [1].

Determining the fractional order in the implementation of these operators is critical to maximize their performance and ensure accurate edge detection results. This process involves selecting the appropriate fractional order, as this factor directly impacts both the quality of detected edges and noise reduction. Despite advances in this field, choosing the optimal fractional order remains a challenge. A new fractional order mask for edge detection based on the fractional Caputo-Fabrizio derivative without singular kernel, exploring values in the range of $0 < \alpha \leq 1$, is proposed in [2], but does not address a specific method to determine the optimal value of the fractional order in its practical applications.

The contribution of this paper consists of a systematic review with the aim of presenting a comprehensive analysis of fractional order determination methods, applied to edge detection. The main approaches used in the literature to adjust the fractional order in edge detection operators are considered, evaluating three aspects: Theoretical foundations, practical applications and computational efficiency. The review focuses on identifying the most promising techniques and proposing new directions for future research. This review aims to provide a clear overview of the

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current state of the art of fractional order methods in edge detection, facilitating the selection of suitable techniques to improve accuracy in image processing.

The article is organized as follows: Section 2 describes the PRISMA methodology used for the systematic review, detailing the data sources, inclusion/exclusion criteria, and study selection process. Section 3 presents the results of the review, including a detailed analysis of the relevant studies in relation to the research questions formulated. In Section 4, key findings, implications for research and practice, and limitations of current methods are discussed. Finally, Section 5 offers conclusions and suggestions for future research.

2. Methods

2.1. Review Method

A systematic approach was adopted for the development of this article based on the guidelines established by [26], [27], and [34].

Seven key research questions were formulated to guide the literature review. Based on these, keywords were selected and specific search equations were defined to filter relevant publications in the chosen databases. After applying six exclusion criteria as filters, 30 relevant articles were selected, whose data were analyzed to answer the questions formulated. This approach ensures an exhaustive, rigorous and relevant systematic review for the research objectives.

2.2. Research Problems and Objectives

Within the framework of the systematic review method, the formulation of research questions is a fundamental step, since they establish the basis for the following stages of the process. The precise formulation of these questions led to the specification of the research objectives. See Table 1.

Table 1. Research Problems and Objectives

ID	Question	Target
RQ1	What are the most frequently used keywords in research on fractional calculus in edge detection?	Identify and analyze the most frequent keywords in the literature on fractional calculus in edge detection.
RQ2	Which are the most productive researchers or authors in academic publications on the application of fractional calculus in edge detection?	Identify the researchers or authors with the highest number of publications on the application of fractional calculus in edge detection.
RQ3	What types of fractional operators are being used in edge detection in images?	Identify the fractional operators used in edge detection in images and their application in image processing.
RQ4	What is the importance of fractional operator kernels in edge detection and how do they compare to traditional kernels?	Describe the importance of fractional operator kernels in edge detection and compare them to traditional kernels.
RQ5	What methods are used to determine fractional order in edge detection operators?	Describe the methods used to determine fractional order in edge detection operators.
RQ6	What are the limitations of the methods used to determine fractional order?	Identify and analyze the limitations and challenges associated with the methods used to determine fractional order in edge detection operators.

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RQ7	What new research paradigms might emerge in the study of fractional calculus applied to edge detection?	Identify new research paradigms that could emerge in the study of fractional calculus applied to edge detection, considering theoretical and practical advances in the field.

2.3. Keywords Definition

The following criteria were used to define the keywords:

i. The analysis of the research questions.

ii. The search in documents in the area of interest to find terms and synonyms.

From the analysis of the research questions and literature review, the keywords needed to identify relevant publications were defined. With these words, specific search equations adapted to each source consulted were formulated.

For the databases *ACM Digital Library, IEEE Xplore, PubMed, ScienceDirect, SpringerLink* and *World Scientific*, the same search equation, described below, was used:

- "fractional calculus" AND "edge detection"

For the *Scopus* database, the keywords used included *"fractional order", "fractional theory", "edge detection techniques"*, and *"edge detection methods"*. In the case of *Google Scholar*, the search was expanded by also incorporating *"fractional kernels"* and *"non-singular kernels"*, adapting the search equation accordingly for each source. The selection of search sources was based on the following criteria:

i. The authors' accessibility to the databases.

ii. The quality and availability of relevant articles for the research.

2.4. Identified Publications

Table 2 shows the number of publications obtained in each search engine after applying the defined equations. This analysis allows us to compare the coverage of the different databases and assess their contribution in terms of the volume of literature relevant to the research.

Table 2. Publications Identified per Source

Search Engine	Number of Publications
ACM Digital Library	9
Google Scholar	50
IEEE Xplore	30
PubMed	1
ScienceDirect	10
Scopus	154
SpringerLink	130
World Scientific	25

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2.5. Selection Criteria

To ensure that the results of our review are as relevant and accurate as possible, the following exclusion criteria were established and applied to the publications reviewed:

- **EC1:** Exclusion of publications that do not correspond exclusively to research articles in indexed scientific journals.
- **EC2:** Articles less than ten years old were considered to ensure that the information is current and relevant.
- **EC3:** Articles written in English were considered.
- **EC4:** Articles that do not provide access to the full text were excluded, preventing an exhaustive evaluation.
- **EC5:** Articles aligned with specific topic of edge detection by fractional calculus were considered.
- **EC6:** Duplicate articles were excluded.

These criteria were defined to ensure the relevance and quality of the sources selected in our review.

2.6. Selected Publications

The application of the search equations in the different search engines generated a total of 409 publications. In the subsequent stage, the selection criteria were applied to obtain the most suitable publications for the article in question. The results of applying the exclusion criteria are shown in Table 3.

2.7. Quality Assessment

The articles selected for this systematic review were subjected to rigorous evaluation based on specific quality criteria. The criteria were developed to ensure that the included studies contributed significant and relevant value to the field of research on the determination of the fractional order in the detection of edges in images.

- **QA1:** Does the title of the article clearly indicate that it is a study on the application of fractional calculus in edge detection?
- **QA2:** Is the article aligned with the main research topic?
- **QA3:** Does the article present a well-organized and coherent structure?
- **QA4:** Does the article address relevant and significant information for ongoing research?
- **QA5:** Is the article written in a clear and precise manner?
- **QA6:** Does the article use and cite reliable and well-known sources in the line of research?

The rigorous evaluation ensured that only studies of the highest quality and relevance were considered in the final analysis, thus guaranteeing the robustness and validity of the conclusions in the review. Figure 1 shows the flow chart resulting from the application of the PRISMA methodology.

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3. Results

Recent studies have explored image edge detection using methods based on fractional calculus, which address the limitations of traditional techniques such as those described in [3]. This work shows that integer-order methods, including Sobel, Prewitt, and Roberts, are generally more sensitive to noise, often leading to less accurate or noisier edge detection results. To overcome this, the authors developed an edge detector based on the Caputo and Caputo-Fabrizio fractional operators. Their findings indicate that a fractional order value of 0.9 provides better results.

Research in [4] propose an active contour model (ACM) focused on edge detection in noisy and inhomogeneous images. By integrating the fractional Caputo-Fabrizio (CF) operator and preprocessing strategies based on local statistics, they achieve contour extraction by suppressing background artifacts. However, its implementation entails a higher computational cost and a high sensitivity to parameter variation.

The proposal of [1] develop fractional masks based on the fractional Atangana-Baleanu integral in the Caputo (ABC) sense, achieving edge detection by flexible selection of the fractional order. Nevertheless, the optimal choice of the fractional parameter α is still a challenge, as it requires specific empirical settings for each image.

In an effort to improve the classical algorithms, [5] introduce fractional operators in Canny, replacing integer-order derivatives with fractional-order operators such as Grünwald-Letnikov (GL), Riemann-Liouville (RL), Caputo (C), Caputo-Fabrizio (CF) and Atangana-Baleanu in the sense of Caputo (ABC). This makes it possible to generate sharper edges and reduce false negatives in regions with complex textures. Similarly, [6] propose a generalized fractional operator based on Grünwald Letnikov, Riemann- Liouville and Liouville-Caputo, optimizing parameters by means of PSNR and Mean Square Error (MSE). However, computational complexity and noise sensitivity are challenges in both approaches.

Table 3. Exclusion Criteria

Search Sources	Number of Publications	EC1	EC2	EC3	EC4	EC5	EC6
ACM Digital Library	9	8	2	2	2	2	2
Google Schoolar	50	40	38	7	7	7	7
IEEE Xplore	30	29	11	11	6	5	5
PubMed	1	1	1	1	1	1	1
ScienceDirect	10	9	6	6	6	3	3
Scopus	154	144	63	62	23	7	7
SpringerLink	130	87	70	70	4	4	4
World Scientific	25	10	10	10	2	1	1
TOTAL	409	328	201	169	51	30	30

Fractional cross-correlation filter (FCCF) is proposed for edge detection in grayscale images in the investigation of [7]. This filter uses a modified cross-correlation equation and an optimized kernel to maximize edge detection. The research shows that the parameter K_p, dependent on the fractional parameter α, influences the number of detected edges. Compared to Sobel and Roberts, FCCF offers better edge detection performance and lower computational complexity. However, it has limitations, such as the inability to detect edges in binary images and the need to manually adjust K_p according to the image texture.

The proposal in [8] presents an approach for the segmentation of water-repellent insulator images, using fractional Atangana-Baleanu in the sense of Caputo operators. The

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study aims to improve edge detection and salt-and-pepper noise in digital images, using fractional masks designed with various discretization techniques. It highlights the flexibility of the fractional parameter α, which allows optimizing edge detection, depending on the type of image analyzed. However, the study also some disadvantages, such as the complexity in the implementation of the fractional operators due to their non-local nature, the need for additional exploration to determine the optimal value of the α parameter and the difficulty in the numerical approximation of these operators.

A fractional differential Prewitt filter based on Asumu's fractional derivative is implemented for edge detection in SARS-COV2 images in the investigation of [9]. The main objective is to develop a method capable of avoiding noise and detecting enriched edge details, with the possibility of adjusting the detection by varying the fractional order α. Experimental results showed that the proposed method outperforms some traditional filters, offering better localization and lower sensitivity to noise. However, the need to adjust α empirically is encountered, which does not guarantee obtaining the best result in all cases.

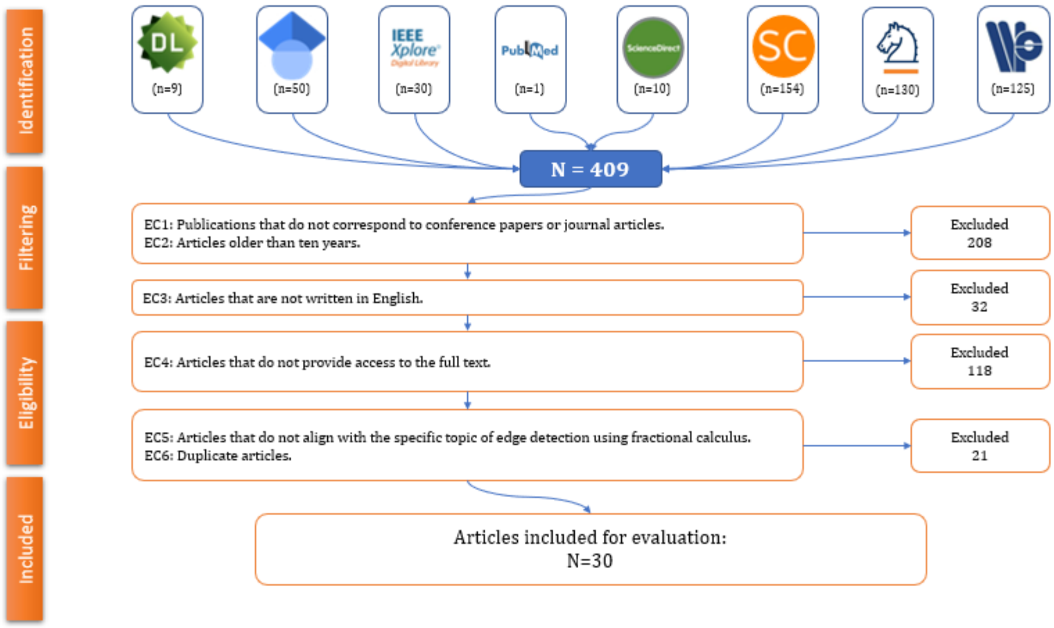

Figure 1. PRISMA flow diagram

In [10] presented the FoGDbED method, which uses fractional-order Caputo-Fabrizio (CF) Gaussian derivatives combined with Gaussian filtering and refinement techniques such as non-maximal suppression (NMS) and thresholding with hysteresis. This method proved to be more efficient than Canny in structure preservation and noise sensitivity reduction, although it requires precise parameter tuning and its theoretical understanding is more complex.

Other studies explored the application of fractional techniques in specific scenarios. In [11] proposed a method based on Caputo-Fabrizio derivatives in the framework of the Classic+NL algorithm, incorporated a coarse-to-fine multiresolution strategy and optimization using Firefly Algorithm (FA) to minimize errors in edge detection. However, the dynamic adjustment of the fractional order entailed a higher computational cost.

An approach for blood vessel segmentation in retinal images is developed using a fractional Gaussian kernel and a fractional Hessian matrix based on Atangana-Baleanu-Caputo in [12]. Despite its high accuracy, the disproportion between vascular and nonvascular pixels affects the correct interpretation of the accuracy metric.

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A fractional global threshold-based edge detector (FGTED) was implemented in [13], which employs the Riemann-Liouville fractional integral operator (RLFI) to improve edge detection without prior training. Compared to classical methods such as Sobel and Canny, this approach achieves higher accuracy and computational time efficiency, although it faces limitations such as the generation of broken edges and false positives under certain conditions.

Research in [14] succeeded in developing an approach inspired by swarm intelligence, combining a fractional-order ant colony algorithm with a fractional differential mask based on the Grünwald-Letnikov operator and the coefficient of variation (FACAFCV). This method improves detection in noisy environments, although its implementation requires precise parameter tuning and entails higher computational cost.

In the same direction, [15] presented an edge detection model based on a fractional-order cumulative gray system, integrating the GM(1,1) model, discrete fractional operators and the discrete wavelet transform (DWT). Their approach allowed preserving details in data-limited scenarios, although computational complexity and the need for parameter tuning remain challenges.

In [16] they proposed two fractional-order gradient masks derived from the Grünwald-Letnikov differential-integral operator. These adaptations of the traditional Sobel filter demonstrated increased robustness to noise and flexibility in their parameterization However, their implementation requires careful adjustment of the fractional order parameter, which increases the computational complexity.

In a similar approach, [17] presented an adaptive two-dimensional fractional differential operator (2D-AFCD) that adjusts differencing according to the local complexity of the image, improving denoising and edge detection. Its versatility and computational efficiency make it suitable for various applications, although its effectiveness may depend on the specific characteristics of the image being analyzed.

Another proposal given by [18] consists of the FOPSTRTV algorithm, which combines fractional-order Phase Stretch Transform (PST) and Total Relative Variation (RTV) to improve image quality. Their method allows for better edge detection and noise reduction, but its high computational cost limits its application in real time.

According to [19] they explore single-pixel image enhancement using fractional operations in the Fourier domain, allowing edge detection without full image reconstruction. Their effectiveness in noise modulation and edge enhancement depends on the fractional order setting, which can affect the ratio of smoothing to detail preservation.

In a similar approach, [20] develop a method based on the Caputo-Fabrizio fractional derivative applied to a scheme similar to the Canny detector. Its implementation requires the construction of fractional digital masks, improving edge detection compared to Canny and Amoako, albeit with higher computational cost and dependence on the fractional order setting.

In [2] they develop an edge detection operator based on the Caputo-Fabrizio fractional derivative, optimized for segmentation of medical images such as mammograms and angiograms. Using an exponential function as a kernel, this approach improves accuracy in identifying relevant structures, albeit with a high computational cost and the need for optimal fractional parameter tuning.

In [21] they implement a method for edge detection in color images based on the Grünwald-Letnikov fractional derivative in multiple directions. Its ability to highlight image gradients improves fine edge detection and its robustness against noise, although it requires parameter tuning for different types of images.

Recently in [22] developed an approach to improve edge detection in skin lesion images by fractional differentiation and classification with neural networks. Although it

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demonstrates high accuracy, it faces challenges such as dataset dependency and risk of overfitting.

According to [23] the proposed algorithm based on fractional computation to improve image quality on mobile devices optimizes illumination and contrast correction. Its implementation offers low computational complexity, although it may generate edge over-enhancement and noise in certain scenarios.

In [28] developed an edge detection method using Grünwald-Letnikov fractional derivatives, avoiding prior smoothing and maximizing Pearson's correlation for optimal fractional parameter (α) fitting. This approach reduced noise and improved edge accuracy with low computational cost, although its fit remains challenging.

According to [29] they propose a method for the detection of arteries in coronary angiograms, incorporating the Grünwald-Letnikov fractional derivative in Frangi's method to improve the segmentation of tubular structures. This approach optimizes the detection of blood vessels using a fractional Hessian matrix, although it faces challenges in the selection of the fractional order (α) and in the computational demand of fractional convolutions.

In [30] they implement an edge detector based on the fractional-order Sobel operator, adapting its masks to the fractional Grünwald-Letnikov calculus. Its application in roadblock detection reduces false positives and allows edge thickness adjustment according to the α parameter, although it suffers from discontinuities and high sensitivity to image features.

In the medical context, [31] employ fractional Grünwald-Letnikov filters for the detection of Alzheimer's disease from magnetic resonance imaging (MRI). Their ability to enhance textures and improve edge detection stands out over traditional methods, although their implementation faces computational challenges and the need for careful fractional parameter tuning.

An edge-relevant structural feature (ERSF) detector based on fractional-order Gaussian derivatives with the Caputo-Fabrizio definition was proposed by [32]. Combining Gaussian filtering and Canny-type detection techniques, this method improves selectivity and robustness to noise, although it requires precise parameter tuning to ensure optimal performance.

From the above, it can be deduced that the panorama of advances in edge detection using fractional calculus highlights the potential of these methods to overcome the inherent limitations of traditional techniques. However, despite the progress made, crucial questions remain about the effectiveness and practical applicability of fractional calculus in this area.

In order to address these challenges and provide a deeper understanding, the article focuses on answering a series of research questions, presented in Table 1. A summary of the important characteristics of the fractional methods used in the investigated literature are visualized in Tables 4 and 5, respectively.

3.1. Answer to the Research Questions

The answers to the research questions posed in this study are presented, which designed to comprehensively address the trends and challenges in the application of fractional calculus to edge detection in imaging. The analysis has been divided into two categories: the first examines bibliometric trends, providing an overview of the most commonly used keywords and the most productive authors in this field (RQ1 and RQ2). The second category focuses on a detailed analysis of the articles selected for the systematic review, exploring methodological approaches, challenges in determining fractional order, and possible future research (RQ3 to RQ7).

RQ1: What are the most frequently used keywords in research on fractional calculus in edge detection?

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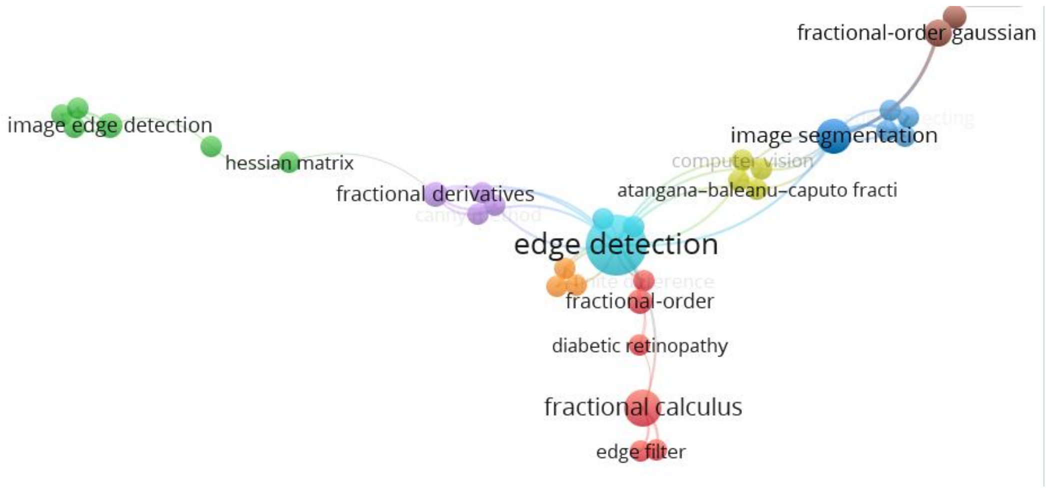
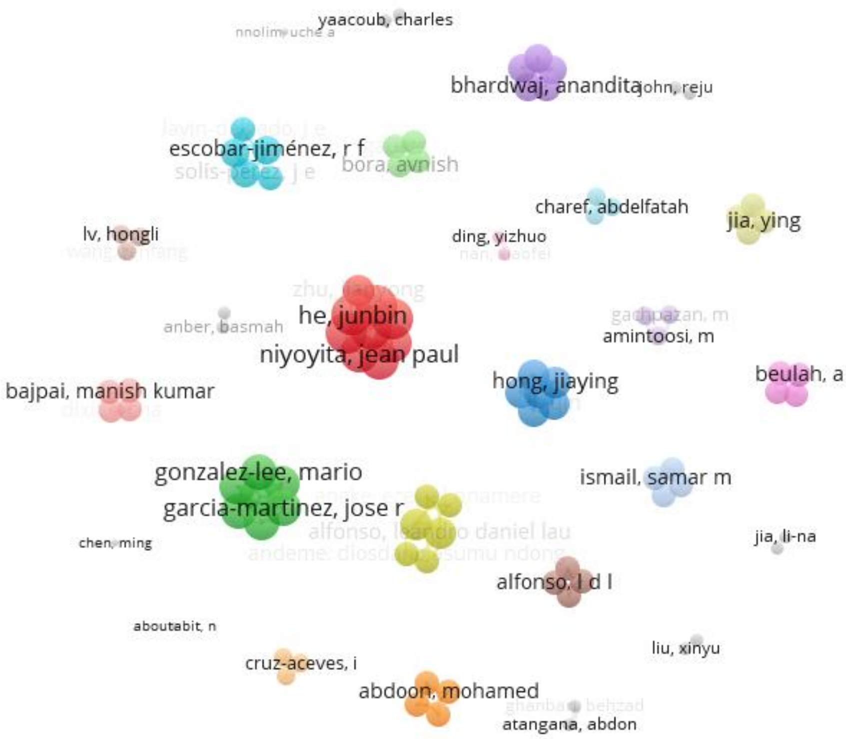
The words that stand out most in the literature on fractional computation applied to edge detection are ’Image Edge Detection’ and ’Detection’. As shown in Figure 2a, these terms stand out due to their high frequency in publications and their relevance in research.

RQ2: Which are the most productive researchers or authors in academic publications on the application of fractional calculus in edge detection?

Based on Figure 2b, which shows the co-authorship density map, it can be seen that authors ‘J. He’, ‘J. Liu’, ‘R. F. Escobar-Jiménez’, ‘J. E. Solis-Pérez’, ‘L. J. Morales-Mendoza’, ‘J. R. García- Martínez’, ‘L. D. Lau Alfonso’ and ‘E. Nnamere Aneke’ lead in terms of number of publications related to fractional calculus in edge detection. Their predominance in the number of publications highlights their high activity and contribution in this field of study.



(a)



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(b)

Figure 2. Density maps: (a) Most frequent keywords on fractional computation applied to edge detection. (b) Authors with the highest production in publications related to this line of research.

RQ3: What types of fractional operators are being used in edge detection in images?

According to [6], the most commonly used fractional operators include the Grünwald-Letnikov, Riemann-Liouville and Liouville-Caputo definitions.

Grünwald-Letnikov: This approach defines the derivative and fractional integral through a discrete limit. The equations for these operators are expressed as follows:

$$ {}_{a}^{GL}D_{x}^{\alpha} f(x) = \lim_{h \to 0} \left(h^{-\alpha} \sum_{k=0}^{\infty} (-1)^{k}\, \alpha k \cdot f(x - kh) \right) \tag{1} $$

$$ {}_{a}^{GL}D_{x}^{-\alpha} f(x) = \lim_{h \to 0} \left(h^{\alpha} \sum_{k=0}^{\infty} \alpha k \cdot f(x - kh) \right) \tag{2} $$

where $nh = x - a$ and $\alpha \in \mathbb{R}$.

This operator is used 32% of the studies, standing out for its ability to handle infinite series and better capture gradual transitions in the images.

Riemann-Liouville: Used by 12% of the studies performed. This operator is preferred when fractional order derivatives are required in problems where the cumulative integral of the past influences the calculation of the present. The corresponding equations are:

$$ {}_{a}^{RL}D_{x}^{\alpha} f(x) = \frac{d^{n+1}}{dt^{n+1}} \int_{a}^{x} (x - \tau)^{n-\alpha} f(\tau)\, d\tau \tag{3} $$

$$ {}_{a}^{RL}D_{x}^{-\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{x} (x - \tau)^{n-\alpha} f(\tau)\, d\tau \tag{4} $$

for $n < \alpha < n+1$. This operator is especially useful in images with soft or fuzzy edges.

Liouville-Caputo: This method, which combines the Riemann-Liouville concepts with the conventional differencing, is used in 24% of the cases. The definitions for the derivative and fractional integral of this operator are:

$$ {}_{a}^{LC}D_{x}^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{x} \frac{f^{(n)}(\tau)}{(x-\tau)^{\alpha+1-n}}\, d\tau, \quad (n-1 \le \alpha < n) \tag{5} $$

$$ {}_{a}^{LC}D_{x}^{-\alpha} f(x) = \frac{1}{\Gamma(-\alpha)} \int_{a}^{x} (x - \tau)^{\alpha-1} f(\tau)\, d\tau \tag{6} $$

This approach is valued for its applicability in imaging where it is crucial to maintain continuity in the derivatives.

A recent approach is the fractional Atangana-Baleanu operator in the Caputo sense (ABC), used by an additional 12% of the studies [1]. This operator is notable for its non-singular kernel and long-range memory. The associated equations are:

$$ {}^{ABC}_{0}D^{\alpha}_{t} f(x) = \frac{B(\alpha)}{1-\alpha} \int_0^t \frac{df}{dx} E_\alpha \left[-\frac{\alpha (t-x)^\alpha}{1-\alpha} \right] dx, \quad 0 < \alpha \leq 1 \tag{7} $$

$$ {}^{ABC}_{0}I^{\alpha}_{t} f(x) = \frac{1-\alpha}{B(\alpha)} f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t f(x)\,(t-x)^{\alpha-1} dx, \quad 0 < \alpha \leq 1 \tag{8} $$

Where $E_\alpha(\cdot)$ denotes the Mittag-Leffler function, and $B(\alpha)$ is a normalization function, where it is satisfied that $B(0) = B(1) = 1$. It is given by:

$$ B(\alpha) = 1 - \alpha + \frac{\alpha}{\Gamma(\alpha)} \tag{9} $$

As shown in Figure 3, this graph compares the different mathematical approaches applied in edge analysis using fractional calculus, where 32% of the cases use the Grünwald-Letnikov definition, 12% employ the Riemann-Liouville definition, 24% adopt the Liouville-Caputo definition, 12% apply the Atangana-Baleanu definition in the Caputo sense, and the remaining 21% resort to other definitions.

It is relevant to note that the paper by [5] highlights that the ABC operator provides superior results by significantly reducing the occurrence of false edges.

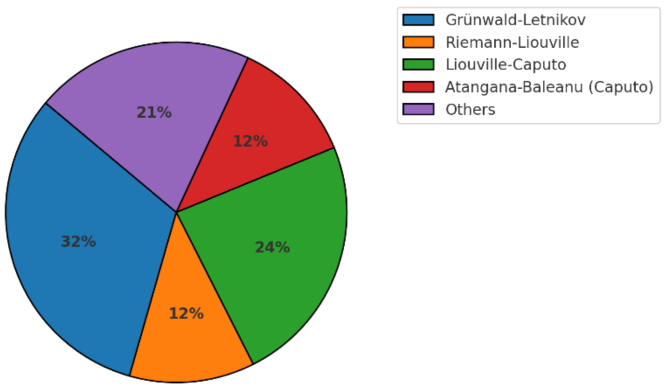

Figure 3. Most common fractional operator definitions applied to edge detection

RQ4: What is the importance of fractional operator kernels in edge detection and how do they compare to traditional kernels?

Fractional operator kernels represent a more flexible and accurate alternative for edge detection compared to traditional operators such as Roberts, Canny, Prewitt and the Laplacian operator. Unlike these deterministic and scale-invariant approaches, fractional operators can model phenomena at multiple temporal and spatial scales, incorporating more complete memory effects and more complex probability distributions. This allows for better detection of subtle structures in images.

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For example, power-law based kernels, employed in fractional Riemann-Liouville-Caputo operators, introduce an artificial singularity that limits their ability to capture memory effects due to their Markovian nature. In contrast, operators with non-singular kernels, such as those proposed by Caputo-Fabrizio and Atangana-Baleanu, employ exponential functions and the Mittag-Leffler law, respectively, allowing for a more accurate and adaptive representation of the dynamic behavior of edges in images, optimizing detection in environments with complex intensity variations [12].

The memory effects in these operators refer to their ability to consider information across the entire range of data rather than being limited to adjacent points [14]. Unlike integer-order operators, which use only local information, fractional operators preserve an inherent memory that maintains the key information of the function from its starting point to the desired time [1].

RQ5: What methods are used to determine fractional order in edge detection operators?

Analysis of the literature shows that there is no single method for determining the optimal fractional order value for edge detection operators, as it varies for different regions of an image [1]. Instead of applying a uniform approach, several studies have employed the peak signal to noise ratio (PSNR) metric to optimize the fractional value. This method consists of selecting an initial value, calculating the PSNR and iteratively adjusting it until its maximum value is reached, which indicates the best approximation to the optimal fractional order. Recent research [5, 6, 8] has validated the effectiveness of this strategy in improving edge detection in various image processing applications.

Other approaches have explored frequency-dependent image regionalization. [13] use thresholding to differentiate between smooth and high-frequency regions, while [24] employ spectral analysis to segment the image into low-frequency and high-frequency areas.

In [17] they propose local variance as an adaptive criterion to adjust the fractional order of the operator depending on the image characteristics, distinguishing between homogeneous regions (low frequency) and areas with textures and edges (high frequency).

RQ6: What are the limitations of the methods used to determine fractional order?

The methods for determining the fractional order in edge detection operators have several limitations. One of the main restrictions is their dependence on a reference image or ground truth to estimate the optimal value of the fractional parameter, which makes them difficult to apply in scenarios without a suitable reference. Without a ground truth, the evaluation of the fractional operator is complicated, since it is not possible to effectively compare its performance.

According to [7], traditional metrics such as Mean Square Error (MSE), Signal-to-Noise Ratio (SNR) and Peak Signal-to-Noise Ratio (PSNR) have proven to be insufficient when edge preservation is crucial, as they do not always accurately reflect the operator’s ability to detect and retain edges [16].

The method proposed by [17] also faces challenges due to the variability of local image features. In the segmentation domain, [25] identifies several limitations in the exclusive use of local variance, such as its high sensitivity

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to noise, subsegmentation in noisy environments, loss of details and fuzzy edges, and difficulties in classifying homogeneous regions.

Although the study by [25] focuses on segmentation and not edge detection, their findings on local variance may be relevant for improving the adaptivity of fractional operators. This highlights the need to develop more robust strategies that reduce the dependence on “ground truth” and improve stability against noise.

Table 4. Characteristics of fractional methods in the literature (Part 1)

No.	Reference	Fractional Operator	Noise Robustness	Parameter Dependence	Degrees of Freedom
1	[33]	GL	No	Yes	1
2	[29]	GL	Not specified	Yes	1
3	[4]	GL	Yes	Yes	3
4	[28]	GL	Yes	Yes	1
5	[30]	GL	Yes	Yes	1
6	[31]	GL	Yes	Yes	1
7	[32]	CF	Yes	Yes	5
8	[1]	ABC	Not specified	Yes	1
9	[5]	GL, RL, C, CF, ABC	Yes	Yes	1
10	[6]	Generalized Operator	Parameter-dependent	Yes	2
11	[7]	RL	Parameter-dependent	Yes	1
12	[8]	ABC	Yes	Yes	1
13	[9]	Asumu - Caputo	Yes	Yes	1
14	[3]	C, CF	Yes	Yes	1
15	[10]	CF	Yes	Yes	6
16	[11]	CF	Yes	Yes	1
17	[12]	ABC	Yes	Yes	3
18	[13]	RL	Not specified	Yes	2
19	[14]	GL	Yes	Yes	10
20	[15]	GM(1,1)+DWT	Yes	Yes	1
21	[16]	GL	Yes	Yes	1
22	[24]	P-Laplace	Yes	Yes	1
23	[17]	2D-AFCD	Yes	Yes	3
24	[18]	FRFT-PST	Yes	Yes	6
25	[19]	GL	Yes	Yes	2
26	[20]	CF	Yes	Yes	1
27	[2]	CF	Yes	Yes	1
28	[21]	GL	Yes	Yes	1
29	[22]	GL, RL	Yes	Yes	1
30	[23]	GL	Not specified	Yes	3

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Table 5. Characteristics of fractional methods in the literature (Part 2)

No.	Computational Complexity	Fractional Order Values Evaluated	Optimization Method	Metrics Used	Adaptive Adjustment by Regions
1	Not specified	[0.1, 0.8]	Empirical	Visual analysis	No
2	Not specified	[0.0, 0.15]	Empirical	ROC	No
3	$O(n \times m)$	Not specified	Empirical	DSC	No
4	Not specified	[-0.229, -0.129]	Correlation maximization	Pearson correlation coefficient	No
5	Not specified	[0, 1]	Empirical	Visual analysis	No
6	Not specified	[0.1, 1.5]	Empirical	PSNR, MSE	No
7	Not specified	[-1, 2]	Empirical	Visual analysis	No
8	Not specified	[0.2, 2]	Empirical	MSE, PSNR	No
9	Not specified	[0.2, 1]	Empirical	MSE, PSNR, SSIM, PR	No
10	Not specified	[0, 1]	Empirical	MSE, PSNR	No
11	Not specified	[0.1, 0.5]	Empirical	GCMSE, EBSSIM, MS-SSIM	No
12	Not specified	[0.5, 2.3]	Empirical	PSNR, ENTROPY	No
13	Not specified	[0.4, 0.51]	Empirical	Not specified	No
14	Not specified	0.9	Empirical	PSNR	No
15	$O(n \times m)$	[-1, 2]	Empirical	SNR	No
16	Not specified	[0, 1]	Firefly optimization	AAE, EPE	No
17	Not specified	[0, 1]	Cuckoo Search (CS) optimization	DCS, ACC, SEN, SPE. FPR, bACC	No
18	$O(n \times m)$	[0, 1]	Empirical	SSIM, F-score, PR, PSNR	No
19	Not specified	[-0.8, 0.8]	Empirical	Recall, Precision, F-measure	No
20	Not specified	Not specified	Not specified	Not specified	No
21	Not specified	[0.1, 1]	Empirical	RMSE, PSNR	No
22	Not specified	Not specified	Adaptative optimization	PSNR, SSIM	Yes
23	$O(n \times m)$	Not specified	Adaptative optimization	PSNR, MSSIM, FOM	Yes
24	Not specified	Not specified	Empirical	Not specified	No
25	Not specified	[-1, 1]	Empirical	SNR	No
26	Not specified	[0.01, 1]	Empirical	MSE, PSNR, SSIM	No
27	Not specified	[0.1, 0.9]	Empirical	MSE, PSNR, PR	No
28	Not specified	[0.1, 1.7]	Empirical	DER, DCR, DCS	No
29	Not specified	0.5	Empirical	Training loss, training acc, test acc, F1-score	No
30	$O(n \times m \times (w-1))$	[0, 1]	Empirical	CEF, PQM, HDI, others	No

RQ7: What new research paradigms might emerge in the study of fractional calculus applied to edge detection?

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The study of fractional computation applied to edge detection is constantly evolving, and driven by theoretical and practical advances, new research paradigms will emerge. These paradigms offer the following innovative perspectives to improve the accuracy and applicability of fractional methods in edge detection.

a) **Adaptive fractional order optimization:** Dynamically adjusting the fractional order according to local image features can improve edge detection. Studies such as [1] and [6] highlight the importance of this approach to achieve higher accuracy and adaptability.

b) **Integration with Machine Learning:** Combining fractional operators with deep neural networks and other machine learning techniques could optimize fractional parameter selection and improve algorithm performance [8].

c) **Multi-scale models:** Developing fractional models that operate at multiple scales would allow for more robust edge representation at different levels of granularity. Approaches such as those of [12] may improve detection in complex images.

d) **Advances in fractional kernel functions:** Exploration of new fractional kernels, such as those based on the Mittag-Leffler function or adaptive kernels, could improve the ability to detect fine details. In [6] and [13] emphasize the need to develop nonsingular and adaptive cores.

e) **Real-time and video applications:** Optimizing fractional methods for moving images and real-time processing could benefit various applications in computer vision, aligning with current trends in dynamic image processing [13].

f) **Exploration of new theoretical paradigms:** Advancing the formulations of fractional calculus, including new definitions of fractional derivatives and fractional integrals with temporal and spatial aspects, could broaden the scope of these methods. In [5] and [30] suggest that theoretical development can provide more robust tools to address challenges in edge detection.

These emerging paradigms present opportunities to advance the field of fractional computation applied to edge detection, offering innovative approaches to overcome current limitations and contribute to the development of advanced image processing techniques.

4. Discussion

The systematic review has identified and synthesized key findings in the study of fractional computation applied to edge detection, highlighting significant advances in theory and practice. It is observed that fractional methods offer greater flexibility and ability to capture fine details compared to traditional operators, which has important theoretical as well as practical implications. From a theoretical perspective, dynamic adjustment of the fractional order and integration with machine learning techniques have been shown to improve the accuracy of edge detection, providing a solid foundation for the development of new adaptive fractional models. These advances could improve the quality of applications in computer vision and real-time image processing. The review also reveals certain limitations, such as the lack of consensus on optimal methods for determining fractional order and the need for more robust approaches for multiscale and video applications. These gaps suggest that future research should focus on optimizing specific fractional techniques and developing models that effectively integrate new theories and

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emerging technologies. The strength of the current approach lies in its ability to integrate diverse methods and results, but identified limitations need to be addressed to advance the field and improve the applicability and efficiency of fractional operators in various practical applications.

5. Conclusions

The systematic review confirmed that fractional computation represents a significant advance in edge detection, overcoming many of the limitations associated with traditional methods. Fractional approaches, by incorporating memory effects and offering flexibility in multiscale analysis, have demonstrated a superior ability to capture complex details and improve accuracy in edge detection. However, despite these advances significant challenges remain, including optimization of fractional order and the lack of standardized methods for its determination. Future research should focus on overcoming these limitations by developing new techniques and integrating advanced theoretical models. In summary, fractional computation offers a promising theoretical and practical framework for edge detection, but continued effort is required to refine existing techniques and explore new applications in image processing and computer vision.

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