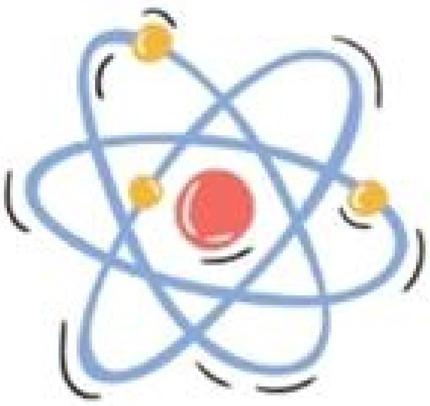

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M. Nirmala Devi, P. Muthukumar

Department of Mathematics, Gobi Arts & Science College, (Affiliated to Bharathiar University, Coimbatore), Gobichettipalayam – 638 453, Tamil Nadu, India.

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M. Nirmala Devi[1], P. Muthukumar[2*]

Department of Mathematics, Gobi Arts & Science College,
(Affiliated to Bharathiar University, Coimbatore)
Gobichettipalayam – 638 453, Tamil Nadu, India.
[1]nirmala1479@gmail.com, [2]muthukumardgl@gmail.com

***Corresponding Author**

ABSTRACT: This work investigates the global chaos synchronization of identical conservative chaotic systems using sliding mode control. The stability results obtained for the complete synchronization of identical conservative chaotic systems are established utilizing Lyapunov stability theory. Numerical simulations are used to illustrate and validate the theoretical findings.

KEYWORDS: Chaos; Sliding mode control; Synchronization.
MSC 2010 CLASSIFICATION: 34H10; 34D06

1. INTRODUCTION

In recent years, Chaos synchronization has gained popularity in nonlinear science due to its potential and practical applications in a wide range of domains. Pecora and Carroll [1] proposed the idea of synchronizing two chaotic systems, whereas [2] presented the synchronization of two similar chaotic systems with differing initial circumstances.

The synchronization problem is a typical challenge in controlling chaotic systems. This involves ensuring synchronous behavior between two systems with comparable chaotic attractors but differing initial conditions. It is important to assure the synchronous behavior of two mutually connected chaotic systems with identical different chaotic attractors. Synchronization of chaotic systems using linear and non-linear feedback controllers have been investigated in [3]. An active control approach has been established and necessary conditions have been derived for synchronizing hyperchaotic sys-tems in [4]. Synchronization of chaotic systems using Lyapunov stability theory has been studied in [5, 6]. Predefined time synchronization method between three dimensional system and multi-wing hyperchaotic system was constructed in [7].

Sliding mode control has recently gained popularity as a robust nonlinear control approach. SMC's main advantages are its tolerance to parameter uncertainty and invariance to unknown disturbances. In [8], the synergetic control theory's forced sliding mode was used to construct synchronization control methods. The dynamic sliding mode control has been investigated for chemical processes in [9]. The fast global fixed-time terminal sliding mode control for the synchronization problem of a generalized class of nonlinear perturbed chaotic systems has been investigated in [10]. Integral Terminal sliding mode controller has been designed for synchronizing the angular velocities of chaotic satellite systems in [11]. Further, the sliding mode controller plays a more vital role in the field of secure communication, see ([12, 13, 14]).

This research suggests using a sliding mode controller to synchronize two identical conservative chaotic systems because of its robustness. Simulation results are presented to show the effectiveness of the proposed controller. Synchronization using the sliding control method is investigated and concluded that this method is an effective method for synchronizing chaotic systems based on synchronization cost and error.

The remainder of this paper is organized as follows: A description of the conservative chaotic system is presented in Section 2. Section 3 presents the design of a sliding mode controller to synchronize two identical conservative chaotic systems; the developed theory is validated through simulation. The paper is concluded in Section 4.

2. THE CONSERVATIVE CHAOTIC SYSTEM

Consider the three dimensional conservative oscillator [15] is

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -ax + yz \\ \dot{z} &= x^2 - by^2 \end{aligned} \tag{1}$$

where x, y, z are the state variables and a and b are parameters of the system (1).

The conservative oscillator system (1) exhibits chaos when $a = 1$ and $b = 0.68$. The chaotic attractor corresponding to the system (1) is shown in Figure. 1 with different phase portraits. Also, the system has symmetry by changing (x, y, z) to $(-x, -y, z)$, which is shown in Figure. 2.

Consider the system (1) as a master conservative system and the slave conservative system can be written as follows:

$$\begin{aligned} \dot{x}_1 &= y_1 + u_1 \\ \dot{y}_1 &= -ax_1 + y_1 z_1 + u_2 \\ \dot{z}_1 &= x_1^2 - by_1^2 + u_3 \end{aligned} \tag{2}$$

where u_1, u_2 and u_3 are controllers to be determined later.

Let $e_1 = x_1 - x, e_2 = y_1 - y$ and $e_3 = z_1 - z$ be the error variables. Then the error system of (1) and (2) can be derived as follows:

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1 \\ \dot{e}_2 &= -ae_1 + y_1 z_1 - yz + u_2 \\ \dot{e}_3 &= x_1^2 - x^2 - b(y_1^2 - y^2) + u_3 \end{aligned} \tag{3}$$

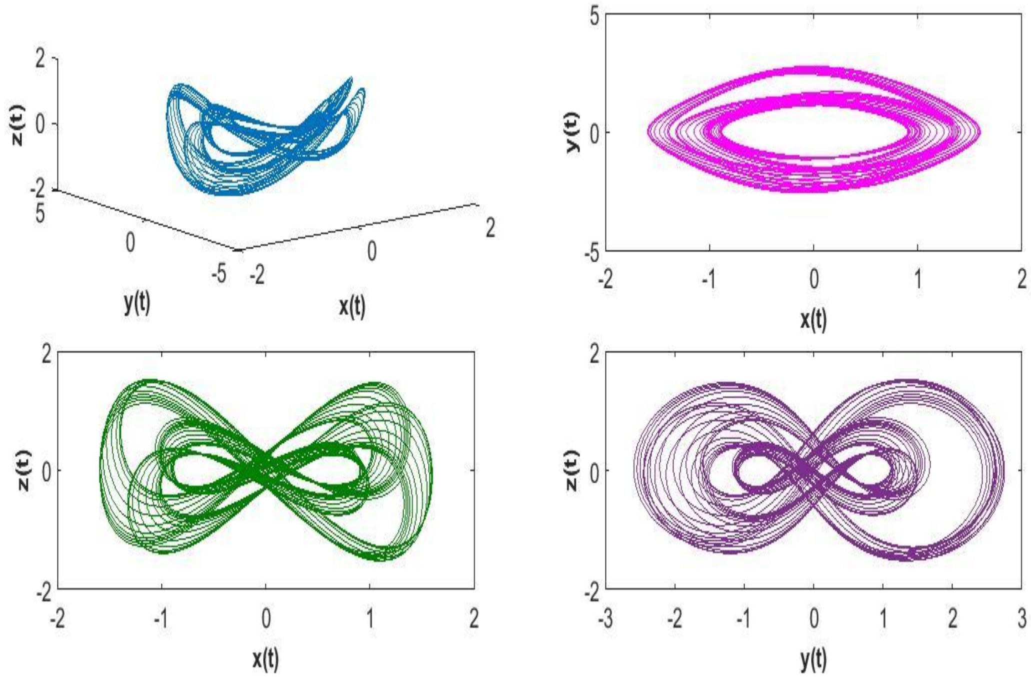

Figure. 1. Chaotic attractor corresponding to the system (1)

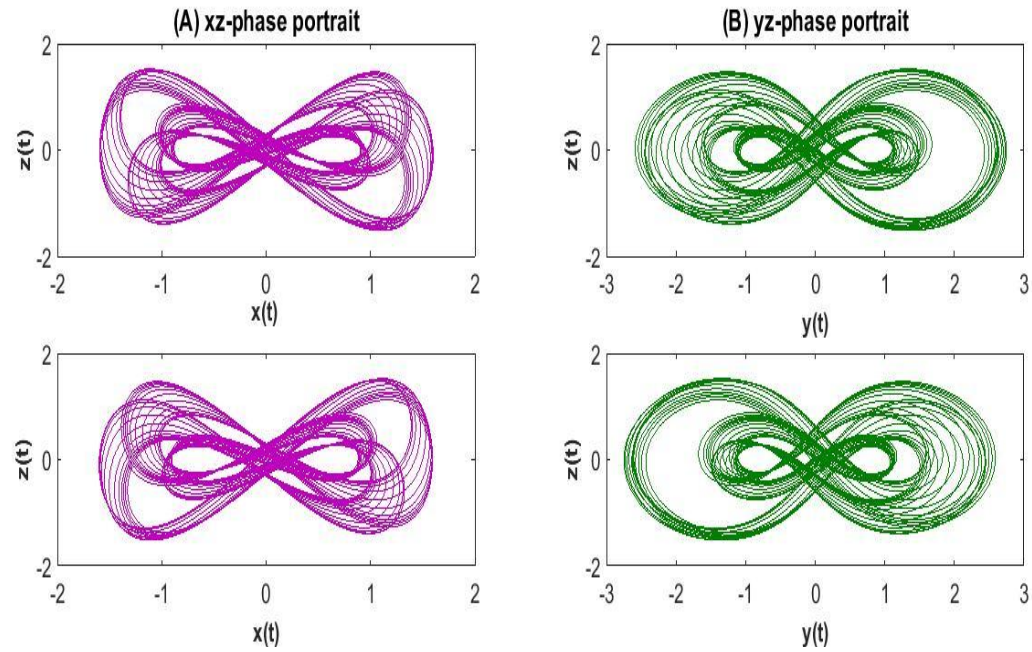

Figure. 2. Symmetric chaotic attractor corresponding to the system (1) : (A) x-z phase portraits (B) y-z phase portraits with initial conditions (0.57,0.99,-0.71)(top) and (-0.57,-0.99,-0.71)(bottom) respectively

3. DESIGN OF SLIDING MODE CONTROLLER AND SYNCHRONIZATION

The objective of this section is to design a sliding mode control for the global synchronization of two identical conservative chaotic systems to force the errors to asymptotically converge to zero as time t tends to infinity.

3.1. Sliding mode controller design

The first step in designing a sliding mode controller is to choose the sliding surface. Let the sliding surface S be such that:

$$S = e_1 + e_2 + e_3 . \tag{4}$$

Theorem 3.1. The systems (1) and (2) will approach global and exponential asymptotical synchronization with the following linear control law:

$$\begin{aligned} u_1 &= -\beta_1 e_1 - \gamma_1 sgn(e_1) \\ u_2 &= -(y_1 z_1 - yz) - \beta_2 e_2 - \gamma_2 sgn(e_2) \\ u_3 &= -(x_1^2 - x^2) - b(y_1^2 - y^2) - \beta_3 e_3 - \gamma_3 sgn(e_3) \end{aligned} \tag{5}$$

where β_i's and γ_i's are positive feedback gains which will be estimated in order to achieve synchronization.

Proof.

The time derivative of sliding surface is

$$\dot{S} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 .$$

Using (3) and (5), we obtain

$$\begin{aligned} \dot{S} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1\left(e_2 - \beta_1 e_1 - \gamma_1 sgn(e_1)\right) + e_2\left(-ae_1 + (y_1 z_1 - yz) - (y_1 z_1 - yz) - \beta_2 e_2 - \gamma_2 sgn(e_2)\right) \\ &+ e_3\left((x_1^2 - x^2) - b(y_1^2 - y^2) - (x_1^2 - x^2) - b(y_1^2 - y^2) - \beta_3 e_3 - \gamma_3 sgn(e_3)\right) \\ &= e_1 e_2 - \beta_1 e_1^2 - e_1\gamma_1 sgn(e_1) - ae_1 e_2 - \beta_2 e_2^2 - e_2\gamma_2 sgn(e_2) - \beta_3 e_3^2 - e_3\gamma_3 sgn(e_3) \\ &= (-a+1)e_1 e_2 - (\beta_1 e_1^2 + \beta_2 e_2^2 + \beta_3 e_3^2) - (e_1\gamma_1 sgn(e_1) + e_2\gamma_2 sgn(e_2) + e_3\gamma_3 sgn(e_3)). \end{aligned} \tag{6}$$

Consider the Lyapunov candidate function as

$$V = \frac{1}{2} S^2 \tag{7}$$

Then the time derivative of V can be written as

$$\begin{aligned} \dot{V} &= S\dot{S} \\ &= (-a+1)e_1 e_2 - (\beta_1 e_1^2 + \beta_2 e_2^2 + \beta_3 e_3^2)S - (e_1\gamma_1 sgn(e_1) + e_2\gamma_2 sgn(e_2))S + (e_3\gamma_3 sgn(e_3))S \\ &< 0. \end{aligned} \tag{8}$$

For every $S \neq 0$, $\dot{V} < 0$. Based on Lyapunov's stability theory, $\lim_{t\to\infty} \| e_i(t) \| = 0, i = 1,2,3$. Thus, the synchronization between systems (1) and (2) is achieved successfully.

3.2. Numerical simulations

Choose the value $\beta_1 = 10$, $\beta_2 = 15$ and $\beta_3 = 10$. Then the synchronized chaotic attractor between master system (1) and slave system (2) is depicted in Figure. 3 and their corresponding state trajectories are depicted in Figure. 4. Further, the time variation of synchronization error between (1) and (2) is shown in Figure. 5.

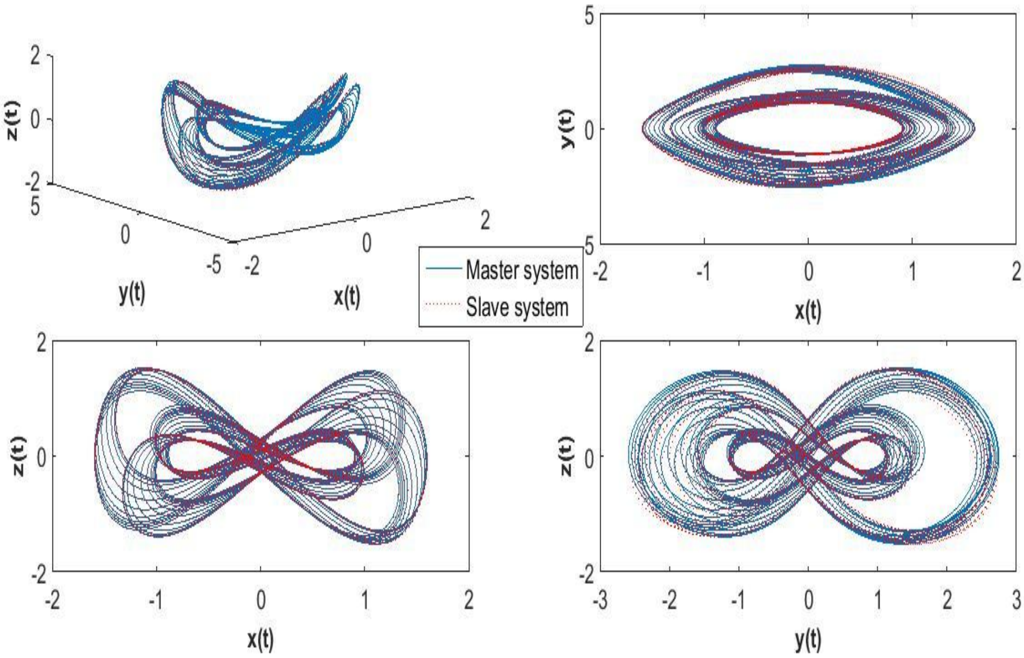

Figure.3. Different phase portraits of synchronized chaotic attractors

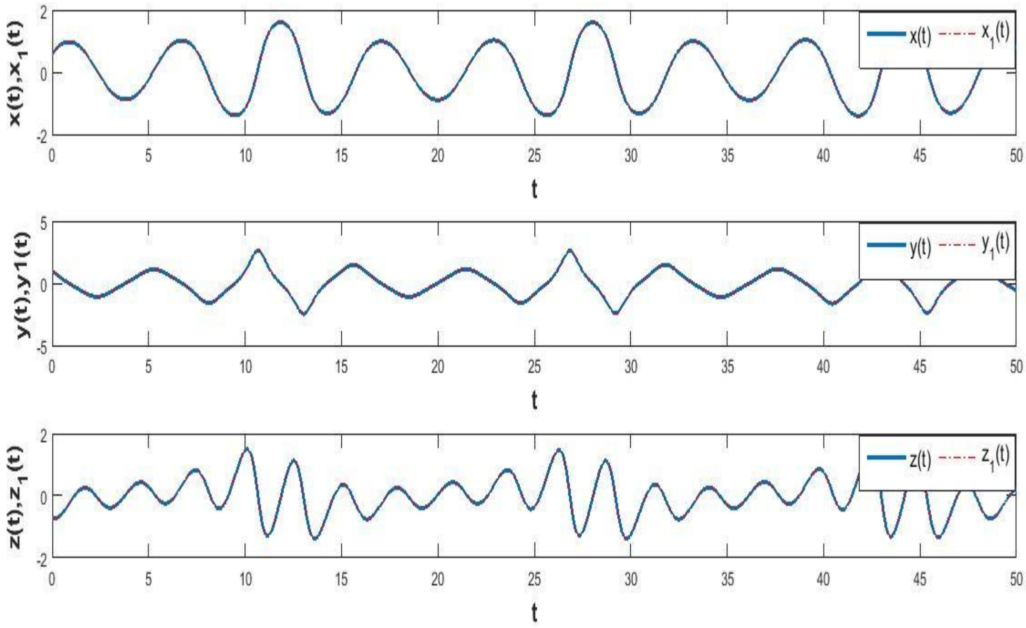

Figure. 4. The evolution of the synchronized states using sliding mode control

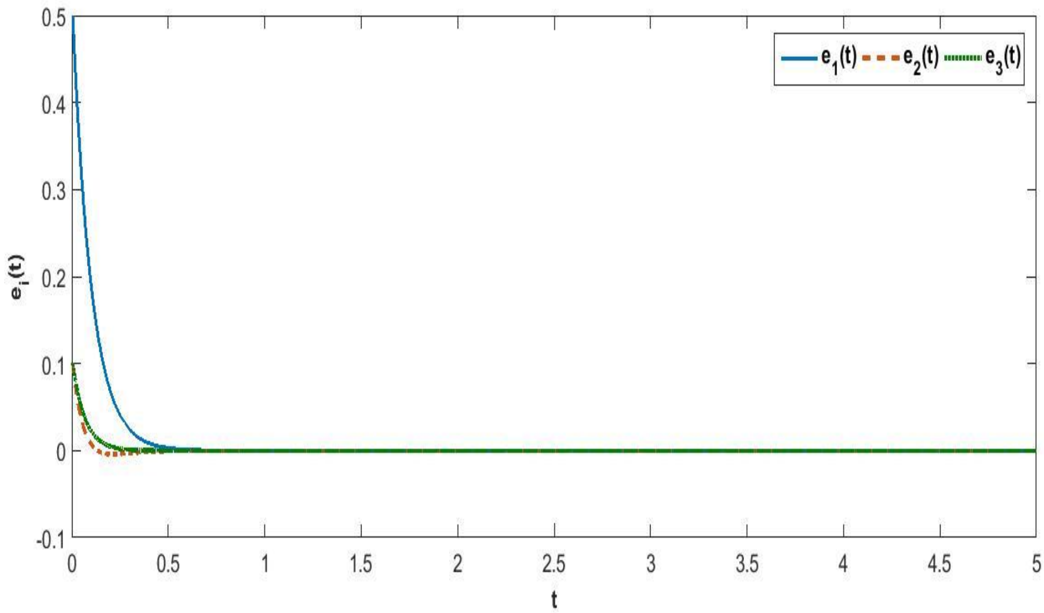

Figure. 5. Time variation of the synchronization error states

4. CONCLUSIONS

In this work, a sliding mode controller to achieve master-slave synchronization in conservative chaotic systems has been developed. The suggested controller assures that the errors between the states of the master and slave systems approach zero as time progresses to infinity. Synchronization utilizing the sliding mode control approach has been found to be more suited and efficient than nonlinear control because of the lower synchronization cost and error. The simulation results clearly demonstrate that the suggested control strategy can synchronize the master and slave systems when they start with differing initial conditions.

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