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Abstract: As we have shown in this letter, frustration in spin glass gives rise to a special kind of Berry connection when a spinor is carried parallel on the surface of the frustrated sphere. The spinor's parallel passage's Berry phase is where the curvature of space is achieved. It's useful to recognize when a spun glass customer is dissatisfied. In the frustrated spin system, this Berry phase can achieve the local order parameter that is topologically stable. Space-time curvature is caused by gauge potentials in gauge theory comparable to berry connections.

1. Introduction

The topological constraint that keeps the surrounding spins from attaining a configuration and frustration has played a crucial role. The frozen spin shows its rigidity rather than the spatial arrangement of spin. The interplay of the spins being at conflict with one another due to an underlying problem leads to frustration. It is commonly known that there are many unexpected aspects to spin glass behavior, and that these unpredictable aspects lead to unhappiness [1]. Wen, Wilczek, and Zee [2] have pointed out that the chiral spin states must be stabilized by the frustration. These states are quantum liquids with an energy gap that are topologically ordered and do not contradict P or T [3]. Hatsugai [4] pointed out that the local topological order of the spin liquid is represented by the quantized Berry phase, which is described by. A highly linked spin system's quantum transport depends critically on the spin Berry phase [5]. Moreover, the chiral anomaly in the field theoretical aspect illustrates the net change in spin chirality caused by this phase [6–10]. Comparing the Berry phase with and without frustration is the aim of this study.

2. Theoretical Background

The granular spin glass system's Hamiltonian [11] changes as spins rotate continuously.

$$H = -J_{ij} \sum_{ij} \{\cos(\phi_i - \phi_j)\} \qquad (1)$$

where J_{ij} is a coupling depending on the nature of host (metal, insulator and superconductor etc.) material. The intricate energy disparity of its grain turns into

$$\psi_i = \{\Delta_i \exp(i\phi_i)\}$$

This resembles an XY spin ferromagnet's Hamiltonian.. All domain barriers have the same energy because of short-range interaction [12].

$$\Delta E(C) = \sum_{ij} J_{ij} \cos(\theta_i - \theta_j) \qquad (2)$$

where $(\theta_i - \theta_j)$ is the angle between the two spins at ith and jth site respectively.

When there is little frustration due to the tight correlation between the spinors, J_{ij} almost determines $\cos(\theta_i - \theta_j)$. When frustration is present, there is a weak association that allows J_{ij} and the other neighbors to determine $(\theta_i - \theta_j)$ as well.

The Hamiltonian in eq.(1) of [11] changes in presence of magnetic field

$$H = -J_{ij} \sum_{ij} \cos\{\phi_i - (\phi_j + A_{ij})\} \qquad (3)$$

Where

$$A_{ij} = 2\pi \int_i^j (\vec{A}.\vec{d}l) \qquad (4)$$

is the gauge potential generated by the interaction of two spins. This demonstrates how a magnetic field may irritate someone. We obtain a quantized model by substituting Pauli matrices for the element of the aforementioned spin vectors. The above Hamiltonian in eq. (3) becomes [4]

$$H = (-J) \sum (S_i^* U_{ij} S_j + h.c) \qquad (5)$$

where $U_{ij} = \exp(iA_{ij})$ represents the link gauge degree of freedom and $S_i = \exp(i\phi_i)$, the spin vector respectively. Instead of the exchange bonds, the difference angle A_{ij} is the source of randomization in this instance. The Hamiltonian maintains its invariance even after the local gauge change.

$$S_i' \rightarrow (V_i S_i)$$

$$U_{ij}' \rightarrow (V_i U_{ij} V_j^*)$$

where $V_i = \exp(i\theta_i)$. The corresponding spin is really rotated by an angle θ_i under local gauge transformation [4] applied at the ith site, and each connected link is rotated by a difference of angle $(\theta_i - \theta_j)$, hence the Hamiltonian stays invariant with such transformation. For the conventional XY model the matrix $U_{ij} = \pm 1$ is restricted to the transformation angles ϕ_i up to $(0, \pi)$.

The relative orientation of nearby spins in a frustrated spin system is determined by the remainder of the spin society as well as by their interactions with one another. The frustrated function is the sign of the product of the exchange integral for any closed route in a lattice spin. In this case, the angles ϕ_{ij} ,which correlate to complicated bonds J_{ij}, are handled as continuous variables. Any closed contour's exchange integral equals -1, but for an un-frustrated system, it equals $+1$. The quantity

$$\exp[2\pi i\phi_{ijkl}] = (U_{ij}U_{jk}U_{kl}U_{li}) \qquad (6)$$

is called the frustration function defined for any closed path in the lattice spins [11]. Incorporating the link gauge degree of freedom by $U_{ij} = \exp(iA_{ij})$, the frustration angle becomes

$$\phi_{ijkl} = (A_{ij}A_{jk}A_{kl}A_{li}) = \sum_{ij} A_{ij} \qquad (7)$$

The connection over a closed path measured using Berry phase is comparable to this sum over link gauge degree of freedom $\sum_{ij} A_{ij}$ in the continuum limit.

A frustrated system is described by a chiral spin liquid where the signature of chiral spinor ψ_L or ψ_R may be considered to represent the order parameter by two opposites orientation of helicities [13]. The rotational orientation of chiral fermion about the axis of anisotropy will generate the topological phase of Berry and thus is associated with the chiral symmetry breaking or chiral anomaly [14]. Since the chirality is associated with the angle denoting the rotational orientation of spin direction vector, the change of rotational orientation of the spin vector will correspond to the change in chirality.

In spherical harmonics $Y_l^{m,\mu}$ (representing the spinor) the spin angular part associated with the angle χ is given by $e^{-i\mu\chi}$. Thus when χ is changed to $(\chi + \delta\chi)$, we have, we have

$$i\left[\frac{\partial}{\partial(\chi+\delta\chi)}\right]e^{-i\mu\chi} = i\left[\frac{\partial}{\partial(\chi+\delta\chi)}\right]e^{-i\mu(\chi+\delta\chi)}e^{i\mu\delta\chi} \qquad (8)$$

which implies that the wave function will acquire the extra phase $e^{i\mu\delta\chi}$ due to innitesimal change of the angle to χ to $(\chi + \delta\chi)$. In fact this is equivalent to the gauge transformation of chiral current [14]. When the angle varies along the enclosed route $0 \le \chi \le 2\pi$, the wave function will obtain the phase after one full revolution

$$\exp[(i\mu)\int_0^{2\pi} \delta\chi] = e^{2i\pi\mu} \qquad (9)$$

which represents the spin dependent Berry phase related to the integral of chiral anomaly. This Berry phase is the measure of chiral change of the spinor [6] in terms of chiral anomaly

$$\Phi_B = i(2\pi\mu)$$

$$\mu = -\left(\frac{1}{2}\right)\int \partial_\mu J_\mu^5 d^4x \qquad (10)$$

where μ behaves as the magnetic charge induced by the background magnetic field.

Anisotropic space $3B$ allows μ to assume values $\pm 1/2$, for which $\Phi_B = e^{2i\pi\mu} = e^{\pm i\pi}$ is the necessary Berry phase. In fact, $\mu = 1/2$ corresponds to a single flux quantum in this formalism, which represents a fermion as a scalar particle traveling in the field of a magnetic monopole. We have the phase $e^{i\pi}$ indicating the system as a fermion that can operate as a signature of the local order parameter in frustrated system as a scalar field (particle) traverses a closed path with one flux quantum encapsulated.

With the specific case of $l = 1/2$, $|m| = |\mu| = (1/2)$ for half orbital/spin angular momentum, we can construct from the spherical harmonics $Y_l^{m,\mu}$, the instantaneous eigenstates $|\uparrow, t\rangle$, representing the two component up-spinor as

$$|\uparrow, t\rangle = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} Y_{1/2}^{(1/2),(1/2)} \\ Y_{1/2}^{(-1/2),(1/2)} \end{pmatrix}$$

$$|\uparrow, t\rangle = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta}{2}\exp i\left[(\phi - \chi)/2\right] \\ \cos\frac{\theta}{2}\exp - i\left[(\phi + \chi)/2\right] \end{pmatrix} \qquad (11)$$

Here in the RHS all the parameters θ, ϕ and χ are time dependent. The charge conjugate down-spinor becomes by

$$|\downarrow, t\rangle = \begin{pmatrix} -Y_{1/2}^{-1/2,1/2} \\ Y_{1/2}^{-1/2,-1/2} \end{pmatrix}$$

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$$\left|\downarrow,t\right\rangle = \begin{pmatrix} -Y_{1/2}^{-1/2,1/2} \\ Y_{1/2}^{-1/2,-1/2} \end{pmatrix} = \begin{pmatrix} -\cos\dfrac{\theta}{2}\exp i\left[(\phi+\chi)/2\right] \\ \sin\dfrac{\theta}{2}\exp -i\left[(\phi-\chi)/2\right] \end{pmatrix} \tag{12}$$

In an arbitrary superposition of elementary qubits $|0\rangle$ and $|1\rangle$ the up-spinors becomes [14]

$$\left|\uparrow,t\right\rangle = \sin\left(\frac{\theta}{2}\right)e^{i\phi}\left|0\right\rangle + \cos\left(\frac{\theta}{2}\right)\left|1\right\rangle)e^{-i/2(\phi+\chi)} \tag{13}$$

The time evolution of a two state system is governed by an unitary $SU(2)2\times 2$ transformation matrix $U(g)$ as follows

$$U(g) = \begin{pmatrix} \alpha & (-\beta)^* \\ \beta & \alpha^* \end{pmatrix} \tag{14}$$

Where $|\alpha|^2 + |\beta|^2 = 1$ with $\left|g\right\rangle = U(g)\left|0\right\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$.

These states $\left|\uparrow,t\right\rangle$ and $\left|\downarrow,t\right\rangle$ can be generated by the unitary matrix $U(\theta,\phi,\chi)$

$$U(\theta,\phi,\chi) = \begin{pmatrix} \sin\left(\dfrac{\theta}{2}\right)\exp i(\phi-\chi)/2 & -\cos\left(\dfrac{\theta}{2}\right)\exp i(\phi+\chi)/2 \\ \cos\left(\dfrac{\theta}{2}\right)\exp -i(\phi+\chi)/2 & \sin\left(\dfrac{\theta}{2}\right)\exp -i(\phi-\chi)/2 \end{pmatrix} \tag{15}$$

from the basic qubits $|0\rangle$ and $|1\rangle$ as follows

$$\left|\uparrow,t\right\rangle = \{U(\theta,\phi,\chi)\left|0\right\rangle, \left|\downarrow,t\right\rangle\} = U(\theta,\phi,\chi)\left|1\right\rangle \tag{16}$$

The holonomy/Berry phase that we see in equation (9) is caused by the fermion's rotation around its quantization axis, which has a fixed helicity. The parameter χ is responsible for displaying the topological characteristics of the chiral fermion [13]. It makes intuitive sense that the fermion acquires this phase by making a single twist at a fixed point on the extended sphere, causing χ to vary exclusively at constant θ,ϕ. A potential consequence of the modification of all three factors might be another Berry phase. It appears that there is no approximation technique that can explain the link between the topological quantum number and the Berry phase.

The Berry phase has a similar connection with chiral anomaly in the setting of symplectic structure deformation in the coherent state representation of a quantized spinor [14]. The single quantized up spinor acquires the geometrical phase along a confined route

$$\gamma \uparrow = i\oint \langle \uparrow, t|\nabla|\uparrow, t\rangle (d\lambda) \qquad (17)$$

$$= i\oint \langle 0|U\dagger dU|0\rangle .(d\lambda) \qquad (18)$$

$$= i\oint A_{\uparrow}(\lambda)(d\lambda) \qquad (19)$$

$$= \frac{1}{2}(\oint d\chi - (\cos\theta)\oint d\phi \qquad (20)$$

$$= \oint A(R)\left(\frac{dR}{dt}\right).dt \qquad (21)$$

$$= i\oint L_{eff}^{\uparrow}\, dt \qquad (22)$$

An extra degree of freedom, denoted as χ, is inserted in the two component spinor z, which is where this effective Lagrangian originates.

$$L_{eff} = iz^{+}\left(\frac{dz}{dt}\right) \qquad (23)$$

This effectively addresses the addition of a certain internal structure to the canonical system. The additional variable behaves as a Hopf fraction and functions as a gauge degree of freedom. In this case, the internal variable functions as a gauge extending the manifold, and the Berry Phase is a solid angle subtended around the quantization axis [15].

$$\gamma \uparrow = i\frac{1}{2}(\oint d\chi - (\cos\theta)\oint d\phi) \qquad (24)$$

$$= (i\pi)(1 - \cos\theta) \qquad (25)$$

The first component in the preceding equation, which is created by changing alone, is equal to the Berry Phase found in equation (9). In actuality, a quantized spinor obtains this BP in terms of the aforementioned solid angle in the extended sphere parameterized by θ, ϕ and χ.

For conjugate state, the down spinor becomes

$$\left|\downarrow, (t)\right\rangle = (-\cos\left(\frac{\theta}{2}\right)\left|0\right\rangle + \sin\left(\frac{\theta}{2}\right)e^{-i\phi}\left|1\right\rangle)e^{i/2(\phi+\chi)} \qquad (26)$$

giving rise on similar manor the Berry phase over the closed path

$$\gamma\downarrow = -(i\pi)(1-\cos\theta) \quad (27)$$

The fermionic or the antifermionic nature of the two spinors (up/down) can be identified by the maximum value of topological phase $\gamma\uparrow/\downarrow = (\pm\pi)$ at an angle $\theta = \left(\frac{1}{2}\right)$. For $\theta = 0$ we get the minimum value of $\gamma\uparrow = 0$ and at $\theta = \pi$ and at no extra effect of phase is realized.

3. Conclusion

Quantized Berry phases, represented as 0 or π, have been seen by Hatsugai [16] to be able to reflect locally their topological order and so be able to classify the frustrated spin systems. With weak exchange couplings, the Berry phases are π, and with high exchange couplings, 0, in a system with a Heisenberg spin chain or frustration. There exist two quantum liquids with different topological phases, which is in line with the adiabatic principle.

4. Discussion

The Berry phases in equations (25) and (27), in the absence of local frustration, represent the solid angle created by the parallel transport of the quantized spinor. The reunion of the starting and finishing points is the outcome of transporting a spinor parallel along a closed path. The transmitted quantized spinor misaligns the beginning and finishing locations because to frustrations in the glassy system.Because the spin direction is unpredictable, the route that the spinor creates is not closed, and the gauge-representing fiber connects the beginning and finishing points by default. The frustrated sphere's geometry gives rise to several types of Berry connections. Our goal is to locate that special Berry phase in glass with frustrated spin.

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