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PRIME CORDIAL LABELING OF HEAWOOD GRAPH, PAPPUS GRAPH AND TUTTE–COXETER GRAPH

J. Vasanthi[1], N. Ramya[2]
[1]Research Scholar, [2]Professor
Department of Mathematics
Bharath Institute of Higher Education & Research,
Chennai, Tamil Nadu, India

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J. Vasanthi[1], N. Ramya[2]
[1]*Research Scholar*, [2]*Professor*
Department of Mathematics
Bharath Institute of Higher Education & Research, Chennai, Tamil Nadu, India
[1]vasanthij1977@gmail.com, [2]drramyamaths@gmail.com

Abstract: *The prime cordial labeling of a graph is defined as* $f: V \rightarrow \{1,2,3,\ldots,|V|\}$ *which is a bijection such that the labels for every edge assigned to 1 if G.C.D* $(f(u), f(v)) = 1$ *and assigned to 0. if G.C.D* $(f(u), f(v)) > 1$ *then* $|\ e_f(0) - \ e_f(1)| \leq 1$. *A prime cordial graphs are the one that allows for prime cordial labelings. This study examines whether the Heawood graph, Pappus graph and Tutte-Coxeter graphs allow for prime cordial labeling.*

Keywords: Heawood graph, Pappus graph, Tutte-Coxeter graphs, prime cordial labeling

1. Introduction

Finite and undirected graphs were used in this paper. An integer is assigned to either or both of the graph's vertices and edges, depending on the circumstances [1]. The mid-1960s saw the first introduction to the labelings of graphs. An extensive bibliography and a dynamic analysis of several graph labelling problems are provided by J.A. Gallian [2][3]. A graph with a defined function of this type is referred as a vertex-labeled graph [4]. In formal terms, a vertex labeling for a graph G = (V, E) is a function of V to a set of labels. An edge labeling is also a function of E to a set of labels is defined as an edge-labeled graph [5][6]. In this we used prime cordial labelings [7][8] for Heawood graphs, Pappus graphs and Tutte-Coxeter graphs [9].

2. Preliminaries

2.1. Definition

If $f: V \rightarrow \{1,2,3,\ldots,|V|\}$ is a bijection for any edge, if G.C.D $(f(u), f(v)) > 1$ ten the labels are assigned to 0 and if G.C.D $(f(u), f(v)) = 1$ then the labels are assigned to 1 therefore $|\ e_f(0) - e_f(1)| \leq 1$.

2.2. Definition

The undirected Heawood graph, named after Percy John Heawood, has 21 edges and 14 vertices.

2.3. Definition

With 27 edges and 18 vertices, the pappus graphs are the 3-regular, bipartite, undirected graphs. This graph has a girth of six. It bears the name of the ancient Greek mathematician Pappus of Alexandria.

2.4. Definition

The Tutte–Coxeter graph, also known as the Cremona–Richmond or the Tutte eight-cage graphs, in the academic field of graph theory, it has a 3-regular graph having 30 vertices and 45 edges.

3. Main Results

3.1. Theorem

Heawood graph is a prime cordial graph.

Proof:

Assume HG be the Heawood graph which has 21 edges and 14 vertices. Edges are $\{V_r V_{r+1}, 1 \leq r \leq 3\} \cup \{V_r V_1, r = 14\} \cup \{V_r V_{r+9}, r = 1,3,5\} \cup \{V_r V_{r+5}, r = 2,4,6,8\}$.

Define $\boldsymbol{f}$: V(HG)⟶ {1,2,3,4,5……….13,14} by

$\boldsymbol{f}(V_r) = 2r, 1 \leq r \leq 7$

$\boldsymbol{f}(V_r) = r-1, r = 8$

$\boldsymbol{f}(V_r) = r+1, r = 10$

$\boldsymbol{f}(V_r) = r-2, r = 9,11$

$\boldsymbol{f}(V_r) = r-9, r = 12,14$

$\boldsymbol{f}(V_r) = r, r = 13$.

In context of the established labeling pattern mentioned above, such that

$e_f(1) = 10$ and $e_f(0) = 11$, $|e_f(0) - e_f(1)| \leq 1$. Therefore HG is a prime cordial labeling.

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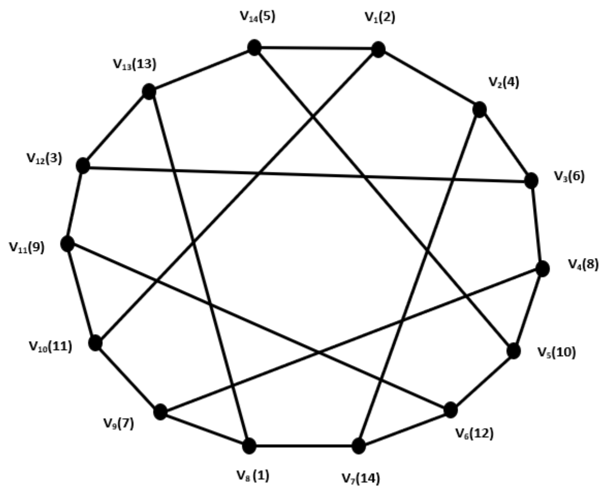

Figure 1. Heawood Graph- Prime cordial labeling

3.2. Theorem

A prime cordial graph is the Pappus graph.

Proof:

A total of 27 edges and 18 vertices are in the Pappus graph (PG). Edges are $\{V_r V_{r+1}, 1 \leq r \leq 17\}$ $\{V_{r-7} V_r, r = 8,10\} \cup \{V_{r-11} V_r, r = 13,15\} \cup \{V_{r-9} V_r, r = 17\} \cup \{V_{r-13} V_r, r = 18\}$.
Define f: V(PG)⟶ {1,2,3,4,5……….,17,18} by

$f(V_r) = 2r - 1, 1 \leq r \leq 5$

$f(V_r) = r+1, r = 7$

$f(V_r) = r-6, r = 8$

$f(V_r) = r+4, r = 9$

$f(V_r) = r, r = 10$

$f(V_r) = r+6, r = 11$

$f(V_r) = r + 3, r = 12$

$f(V_r) = r-1, r = 13,15$

$f(V_r) = r+2, r = 14$

$f(V_r) = r-12, r = 16$

$f(V_r) = r-11, r = 17$

$f(V_r) = r-7, r = 18.$

In context of the established labeling pattern mentioned above, clearly

$e_f(1) = 14$ and $e_f(0) = 13$, $\left| e_f(0) - e_f(1) \right| \leq 1$. So PG is a prime cordial labeling.

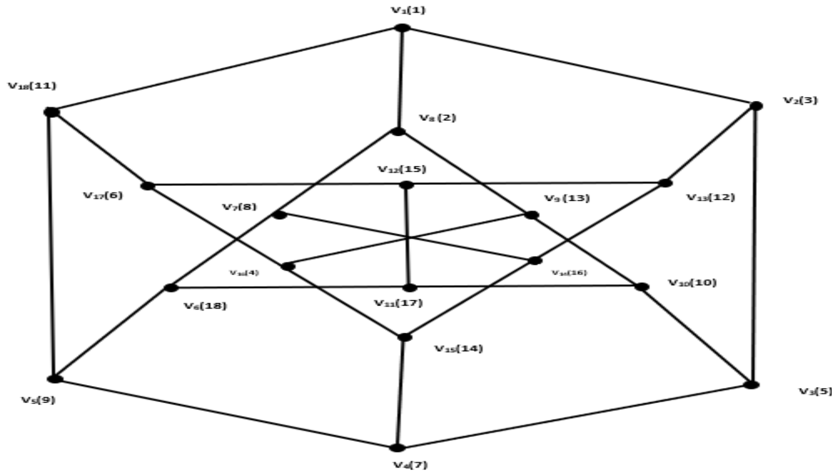

Figure 2. Pappus Graph-Prime cordial labeling

3.3. Theorem

Tutte–Coxeter graph is a prime cordial graph.

Proof:

The Tutte–Coxeter graph (TCG) has 45 edges and 30 vertices. Edges are $\{V_r V_{r+1}, 1 \leq r \leq 29\} \cup \{V_r V_{r+7}, r = 5,11,17,23\} \cup \{V_r V_{r+9}, r = 1,7,13,19\} \cup \{V_r V_{r+13}, r = 2,8,14\} \cup \{V_r V_{r+17}, r = 3,9\} \cup \{V_4 V_{25}\} \cup \{V_6 V_{29}\} \cup \{V_{30} V_1\}$.

Define f: V(TCG)⟶ {1,2,3,4,5……….,29,30} by

$f(V_r) = 2r$, $1 \leq r \leq 7$, r =15

$f(V_r) = r$, r = 11,13

$f(V_r) = r-1$, r = 8,18,28,29

$f(V_r) = r+6$, r =10,12,16

$f(V_r) = r+1$, r = 22,25

$f(V_r) = r-2$, r = 21,27

$f(V_r) = r-4$, r = 19,24

$f(V_r) = r+4$, r = 20

$f(V_r) = r-8$, r = 9

$f(V_r) = r-3$, r = 26

$f(V_r) = r-14$, r = 17,23

$\boldsymbol{f}$ (V_r) = r-9, r = 14,30.

We have $e_f(1) = 23$ and $e_f(0) = 22$ in the context of the previously indicated established labelling pattern, $|\ e_f(0) - e_f(1)| \leq 1$. The Tutte-Coxeter graph is therefore a prime example of cordial labelling.

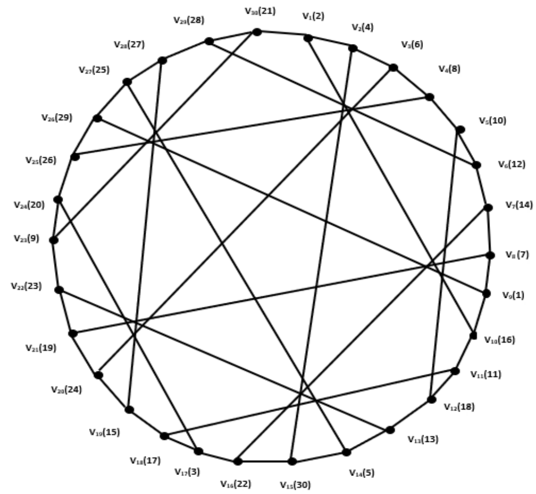

Figure 3. Prime cordial labeling Tutte–Coxeter Graph

4. Conclusion

Prime cordial labeling for the Heawood, Pappus, and Tutte-Coxeter graphs has been studied in this study. Future discussions regarding the duplication and fusion of the Pappus, Heawood, and Tutte-Coxeter graphs under prime cordial labeling are necessary.

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