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Abstract: *We have shown that the chiral shift in helicity of a quantized spinor, represented by the Berry phase, is only detectable in the presence of frustration. Hatsugai noted that the local topological order of the spin liquid is reflected in the quantized Berry phase represented by and. In the quantum transport of strongly correlated spin systems, the spin Berry phase is essential. Moreover, the chiral anomaly in the field theoretical aspect illustrates the net shift of spin chirality induced by this phase. Comparing the Berry phase with and without frustration is the aim of this study.*

1 Introduction

A spin glass's array of spin-ordering is comparable to a tangent vector's "parallel transport" across a curved surface. During parallel transit, the curvature of space causes successive spin vectors to diverge angularly. This is comparable to the existence of dissatisfaction that prevents parallelism from occurring on its own. This is how dissatisfied plaques are positioned, and frustrated plaques are twisted. It appears that the frustrated sphere's shape results in an extra curvature and a different type of Berry connection. The curvature of space is realized by the Berry phase [1] during the spinor's parallel transit. Realizing dissatisfaction in spin glass is beneficial [2]. This Berry phase in the frustrated spin system can realize the local order parameter with topological stability [3]. Space-time curvature is caused by Berry connections, which are equivalent to gauge potentials in gauge theory. In this connection, we would like to offer a fruitful example of a frustrating time. It should be observed that the frustrated and un-frustrated Berry phases may give rise to the nontrivial matrix Berry phase of two qubit states.

2. Theoretical Background

Murakami et.al [4] pointed out that when an electron hops from site i to j coupled to a spin at each site then the spin wave function is effectively

$$|\chi_i\rangle = t\left(e^{ib_i}\cos\left(\frac{\theta_i}{2}\right), e^{i(b_i+\phi_i)}\sin\left(\frac{\theta_i}{2}\right)\right) \quad (1)$$

The overall phase b_i corresponding to the gauge degree of freedom does not appear as physical quantities. The effective transfer integral t_{ij} is given by [5]

$$t_{ij} = t\langle \chi_i | \chi_j \rangle$$

$$= te^{(b_j - b_i)} \left(\cos\left(\frac{\theta_i}{2}\right)\cos\left(\frac{\theta_j}{2}\right) + e^{i(\phi_i - \phi_j)} \sin\left(\frac{\theta_i}{2}\right)\sin\left(\frac{\theta_j}{2}\right)\right)$$

$$= te^{ia_{ij}} \cos\left(\frac{\theta_{ij}}{2}\right)$$

$$= te^{ia_{ij}} \cos\left(\frac{\theta_{ij}}{2}\right) \qquad (2)$$

where the angle between spins $\vec{S}_i$ and $\vec{S}_j$ is represented by θ_{ij}. The vector potential created by the spins, or phase a_{ij}, is what the hopping electron senses as the Berry phase. It has been noted [1] that the solid angle subtended by the three spins is likewise the total phase produced by an electron hopping along a loop $1 \to 2 \to 3 \to 1$. In the context of quantum spin liquid, where the spins vary quantum mechanically, this a_{ij} quantifies the spin chirality.

Taking into consideration the previous studies [4-6], we go on to determining the Berry phase of a quantized spinor that is situated on the surface of a frustrated sphere. We may think of a frustrated sphere as an extended sphere with a quantized fermion. In an unfrustrated system along a closed route, we have previously analyzed the Berry Phase of the quantized spinor. We learn that the quantized spinor need not trace a closed curve in the presence of local frustration. Therefore, there would be a divergence from the original location. A dierent connection and curvature would emerge from the transport of a spinor in the geometry of a frustrated surface. The transfer integral of the quantized spinor will be

$$\langle \uparrow, t_j | \uparrow, t_i \rangle = e^{(i/2)(\phi_j + \phi_i)} . e^{(i/2)(\chi_j + \chi_i)}$$

$$\left(\cos\left(\frac{\theta_i}{2}\right)\cos\left(\frac{\theta_j}{2}\right) + e^{i(\phi_i - \phi_j)} \sin\left(\frac{\theta_i}{2}\right)\sin\left(\frac{\theta_j}{2}\right)\right) \qquad (3)$$

In comparison with that of eq.(3) by Anderson [5],this above transfer integral t_{ij} can be realized as the product of angle θ_{ij} and the parameter $\chi_j + \chi_i$ is visualizing the change of inclination of helicity of the quantized spinor as it moves from site i to j in this frustrated spin glass.

$$\langle \uparrow, t_j \left| \uparrow, t_i \right\rangle = e^{(i/2)(\phi_j+\phi_i)} . e^{(i/2)(\chi_j+\chi_i)} \cos\left(\frac{\theta_{ij}}{2}\right) \quad (4)$$

For the continuous change of helicity

$$e^{i/2(\chi_j+\chi_i)} = e^{i\left(\frac{\delta\chi}{2}\right)} = e^{i\left(\frac{a_{ij}}{2}\right)}$$

the Berry phase of quantized fermion in the wave function will acquire the phase

$$\exp[i\mu \int_0^{2\pi} \delta\chi] = e^{(2i\pi\mu)} \quad (5)$$

which represents the spin dependent Berry phase related to the integral of chiral anomaly

Hence we can have

$$\langle \uparrow, t_j \left| \uparrow, t_i \right\rangle = e^{(i/2)(\phi_j-\phi_i+a_{ij})} \cos\left(\frac{\theta_{ij}}{2}\right) \quad (6)$$

Where $a_{ij} = (\chi_j + \chi_i)$ represent the difference of inclination of helicity over the virtual closed path. Following the local gauge transformation

$$a_{ij} \to (a_{ij} + \phi_i - \phi_j) \quad (7)$$

it seems that the two equations (2) and (4) are equivalent. As a result, the transfer integral for the quantized spinor is comparable to that in [4-5].

The quantized spinors in a frustrated system obtain distinct Berry connections. A single spinor rotation causes the starting point and the final point to diverge, which logically suggests the open contour on the frustrated sphere. When moving around a closed loop, the only thing that changes are the ϕ values from the $0 \to 2\pi$ values; the little shift in the χ values is represented as a change in chiral gauge because of certain conflicts between the spins that the system's diseases offer.

By the use of spinor in an arbitrary superposition of elementary qubits $\left|0\right\rangle$ and $\left|1\right\rangle$ the up-spinors becomes

$$\left|\uparrow, t\right\rangle = \sin\left(\frac{\theta}{2}\right) e^{i\phi} \left|0\right\rangle + \cos\left(\frac{\theta}{2}\right) \left|1\right\rangle) e^{-i/2(\phi+\chi)} \quad (8)$$

After a few mathematical operations, we are able to acquire the necessary Berry connection for a quantized spinor in the frustrated spin system.

$$\langle\uparrow_j|d|,\uparrow_i\rangle = e^{(i/2)(\phi_j-\phi_i)\cdot} e^{(i/2)(\chi_j-\chi_i)\cdot} \left(\sin\left(\frac{\theta_i}{2}\right)\sin\left(\frac{\theta_j}{2}\right)e^{(i/2)(\phi_i-\phi_j)}d\phi_i - \left(\frac{i}{2}\right)\cos\left(\frac{\theta_{ij}}{2}\right)(d\chi_i + d\phi_i)\right) \quad (9)$$

For conjugate state, the down spinor becomes

$$|\downarrow,(t)\rangle = (-\cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{-i\phi}|1\rangle)e^{(i/2)(\phi+\chi)} \quad (10)$$

So the Berry connection is

$$\langle\downarrow_j|d|,\downarrow_i\rangle = e^{i/2(\phi_i-\phi_j)\cdot} e^{i/2(\chi_i-\chi_j)\cdot} \left(\frac{i}{2}\cos\frac{\theta_{ij}}{2}(d\chi_i + d\phi_i) - i\sin\frac{\theta_i}{2}\sin\frac{\theta_j}{2}e^{i(\phi_j-\phi_i)}d\phi_i\right) \quad (11)$$

Because the final site does not correspond with the beginning one, the parallel transport of a spinor via a closed path on a sphere parameterized by θ, ϕ and χ implies an open curve geometrically in a frustrated spin system. After integration over the variation of ϕ by $0 \le \phi \le 2\pi$, the Berry phase for both frustrated up and down spinor in the spin glass system will be obtained.

$$\Gamma_F^{\uparrow} = -2i\pi e^{i/2(\chi_j-\chi_i)}\left(\cos\left(\frac{\theta_{ij}}{2}\right) - \sin\left(\frac{\theta_i}{2}\right)\sin\left(\frac{\theta_j}{2}\right)\right) \quad (12)$$

and

$$\Gamma_F^{\downarrow} = 2i\pi e^{(i/2)(\chi_i-\chi_j)}\left(\cos\left(\frac{\theta_{ij}}{2}\right) - \sin\left(\frac{\theta_i}{2}\right)\sin\left(\frac{\theta_j}{2}\right)\right) \quad (13)$$

These Berry phases $\Gamma_F^{\uparrow}$ or $\Gamma_F^{\downarrow}$ for the frustrated system may be seen as the product of the solid angle of the spinor between the ith and jth site, and the helicity (χ) dependent phase. In this case, we're thinking about a spinor that, frustrated, travels from the ith site to the jth site after just one turn. In the spin system, degeneracy endures. The phase is dependent on both θ_{ij}, the angle between the two spinors, and the individual angles θ_i and θ_j of the spinors.

In absence of local frustration, $\cos\left(\frac{\theta_{ij}}{2}\right)=0$, no spin confict exists indicating the transport of spin vectors ideally parallel. Consequently, the ultimate location aligns with the original selection, resulting in the Berry phase for the unfrustrated system

$$\gamma^{\uparrow}=\left\{i\pi(1-\cos\theta_i)\right\} \qquad (14)$$

and

$$\gamma^{\downarrow}=\left(-i\pi\right)(1-\cos\theta_i)$$

In a frustrated system, $\cos\left(\frac{\theta_{ij}}{2}\right)=\pm 1$, act as a signature of two chirality that may act also as an order parameter in the system. The helicity/internal helicity dependent BP is not evident in an unfrustrated system, even in the presence of a magnetic field, which is one of the very sources of quantization. Only in a frustrated spin glass system, where disorders provide spin conflict to realize both the solid angle and the helicity-dependent phase, is this accomplished.

In a very recent communication [7] we have pointed out that due to frustration in Quantum Hall system, in lowest Landau level $LLL(\nu=1)$ the degenerate two qubit singlet state

$$=\begin{pmatrix}\mu_i & \mu_{ij}\\ \mu_{ji} & \mu_j\end{pmatrix}$$

$$=\left\{(u_i-v_i)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}u_i\\ v_j\end{pmatrix}\right\} \qquad (15)$$

has been recognized as a Hall qubit built using the up-spinor $\left|\uparrow_i\right\rangle=\begin{pmatrix}u_i\\ v_i\end{pmatrix}$. The non-abelian nature of the connection on the Hall surface will remain if $i\neq j$.

$$B_{\uparrow}=\begin{pmatrix}\left\{(u_i^*du_i)+(v_i^*dv_i)\right\} & \left\{(u_i^*du_j)+(v_i^*dv_j)\right\}\\ \left\{(u_j^*du_i)+(v_j^*dv_i)\right\} & \left\{(u_i^*du_i)+(v_i^*dv_i)\right\}\end{pmatrix}$$

$$=\begin{pmatrix}\mu_i & \mu_{ij}\\ \mu_{ji} & \mu_j\end{pmatrix} \qquad (16)$$

The states having degenerate Hamiltonian visualize non-abelian matrix Berry phase in presence of spin conflict during the parallel transport.

$$\gamma_{\uparrow}^{H} = \begin{pmatrix} \gamma_i & (\Gamma_{ij}) \\ (\Gamma_{ji}) & \gamma_j \end{pmatrix} \quad (17)$$

γ_i and γ_j are the un-frustrated Berry Phase for the ith and jth spinor as seen in equation (14) and the off-diagonal Berry Phase Γ_{ij} in eq.(12) arises due to local frustration in the spin system. At the end we like to specify the three different type of Berry phases we have studied so far.

1. A quantized fermion's Berry phase, denoted as $e^{2i\pi\mu}$, may only be created when the parameter χ is varied in relation to helicity [8].

2. It has been demonstrated recently [9–12] that the Berry phase of a quantized spinor cycled across a closed route acquires the typical solid angle $\pi(1-\cos\theta)$, where all parameter variations occur without local frustration.

3. In this short note we have studied the Berry phase of the quantized spinor as

$$\Gamma_F^{\uparrow} = -2i\pi e^{i/2(\chi_j-\chi_i)}\left\{\cos\left(\frac{\theta_{ij}}{2}\right) - \sin\left(\frac{\theta_i}{2}\right)\sin\left(\frac{\theta_j}{2}\right)\right\}$$

in presence of local frustration which is under special condition gives rise the Berry phase of un-frustrated system[13-15].

3. Conclusion

example of a frustrated system with alternating Berry phases for two Landau level filling factors is a quantum Hall state. Reaching the Berry phase results in the realization of gapped energy states. Conversely, the abelian Berry phase is formed by non-degenrate quantum states rotating throughout a cycle. In view of Hwang et al., we know that the pumped charge in the Quantum Hall system flows across a cycle by the singlet states due to the non-abelian matrix Berry phase induced by frustration. A frustration in spin glass is caused by local irregularities in spin transport.

4. Discussion

We have studied the quantized spinor that lives on this frustrated sphere's surface and creates a novel kind of Berry phase. This phase can sometimes transition smoothly into the typical Berry phase. Frustration affects the Berry phase of the non-abelian matrix.

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