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DISLOCATED PHASE AND ANTI-PHASE SYNCHRONIZATION OF CHAOTIC SYSTEMS WITH APPLICATIONS TO SOUNDTRACK ENCRYPTION AND DECRYPTION

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ABSTRACT: *This work investigates the dissipative, equilibrium, and stability properties of a three-dimensional integer-order chaotic system. Designing a suitable controller allows for dislocated phase and anti-phase synchronization of two parallel chaotic systems. A new algorithm for soundtrack encryption and decryption is created based on the presented dislocation synchronization mechanism. Numerical simulations are provided to demonstrate the effectiveness of the theoretical results.*

Keywords: Chaos; Parallel systems; Synchronization; Cryptography
MSC 2010 Classification: 34H10; 76E09; 34D06; 94A60.

1. INTRODUCTION

In 1972, Edward Lorenz employed the term "Butterfly Effect" to introduce chaos theory to the modern world [1-4]. Understanding this principle can help make a complicated system more predictable. Thus, when dealing with a system, you ought to remain aware of all inputs and maintain control over them. Chaotic systems are unstable because they aren't resistant to external disturbances but rather react in substantial manners. In other words, they are partially guided by outside influences rather than rejecting them.

Chaos is one of the most apparent characteristics of nonlinear dynamical systems, whose state variables strongly rely on their starting conditions. This reliance results in diverging behavior in such systems, highlighting the critical significance of careful research into chaotic processes because chaos appears in an extensive variety of domains, including chemical reactions [5,6], dynamo theory [7,8], power systems [9,10], robotics [11,12], economics [13,14], cryptosystems, secure communications [15-18], and so on.

The studies of synchronization by designing suitable control for integer-order chaotic systems have attracted increasing attention among researchers due to their potential applications. Most recently, various techniques and methods have been implemented by researchers to achieve the control and synchronization of integer-order chaotic systems [19-21]. Most recently, the problem of controlling and synchronizing chaos for a new dynamical system has been studied, and sufficient conditions for the synchronization of chaotic systems have been derived in [22]. Chaos synchronization has garnered tremendous worldwide interest in communication systems, which have applications in the encryption and decryption of information for secure communications. An adaptive scheme has been exhibited in [23] for chaos synchronization that solves the problem of security in communications. The multi-scale synchronization scheme of two fractional-order king cobra chaotic systems is described [24]. In [25], a new technique has been suggested for synchronizing two chaotic systems, and that technique has been applied to digital cryptography for sending and receiving messages. The authors in [26], an adaptive terminal sliding-mode control method is proposed for the synchronization of uncertain fractional-order chaotic systems with disturbances.

In this study, we examine dislocated phase and anti-phase parallel synchronization for integer-order chaotic systems. These systems have been synchronized by creating non-linear control functions. Synchronizing chaotic dynamical systems with parallel systems is also an important challenge. From an application point of view, a novel key system for soundtrack cryptographic encryption is offered based on the proposed synchronized chaotic systems that are safer and more powerful than a synchronized key system [27-30]. The encryption and decryption procedures are demonstrated using a numerical simulation, and the security of the recommended soundtrack cryptosystem is examined.

The rest of this study is organized as follows: Problem statements and preliminaries like dissipation and stability of equilibrium points are given in Section 2. The concept of a parallel system for chaotic systems and its dislocated phase and anti-phase synchronization are described in Section 3. The applications of synchronized chaotic systems are discussed, and furthermore, it has been demonstrated that the numerical example works well with the suggested soundtrack cryptosystem in Section 4. Finally, this study concludes Section 5.

2. SYSTEM DESCRIPTION AND ANALYSIS

In this section the dynamical behaviors of a chaotic system are examined, such as dissipativity and stability of equilibriums.
Consider a three dimensional chaotic system [31] with six nonlinear terms as

$$\begin{aligned} \dot{x} &= z \\ \dot{y} &= -z(\alpha y + \beta y^2 + xz) \\ \dot{z} &= x^4 - 0.1x^2y^2 + y^2 - 0.5 \end{aligned} \tag{1}$$

Where $(x, y, z) \in R^3$ are the state variables and α, β are the parameters of the system (1).

The system (1) exhibits chaos when $\alpha = 8$ and $\beta = 3$. The different phase portrait of the chaotic attractor corresponding to the system (1) is shown in Figure.1.

2.1. Dissipativity

A dissipative flow [32] has a property that trajectories are attracted to a bounded region of state space by a strange attractor and its divergence is negative.

For the system (1),

$$\nabla V = \frac{\dot{x}}{x} + \frac{\dot{y}}{y} + \frac{\dot{z}}{z} = -z(\alpha + 2\beta y) < 0 \tag{2}$$

So, the system is dissipative with an exponential contraction rate: $\frac{dV}{dt} = e^{-z(\alpha + 2\beta y)}$

2.2. Equilibrium points and stability

The equilibrium points of the system (1) are calculated by setting $\dot{x} = \dot{y} = \dot{z} = 0$. Then the equilibrium points are
$E_1 = \left(0.8408964i,\ 0,0\right)$, $E_2 = \left(-0.8408964i,\ 0,0\right)$, $E_3 = \left(0.8408964,\ 0,0\right)$ and
$E_4 = \left(-0.8408964,\ 0,0\right)$.

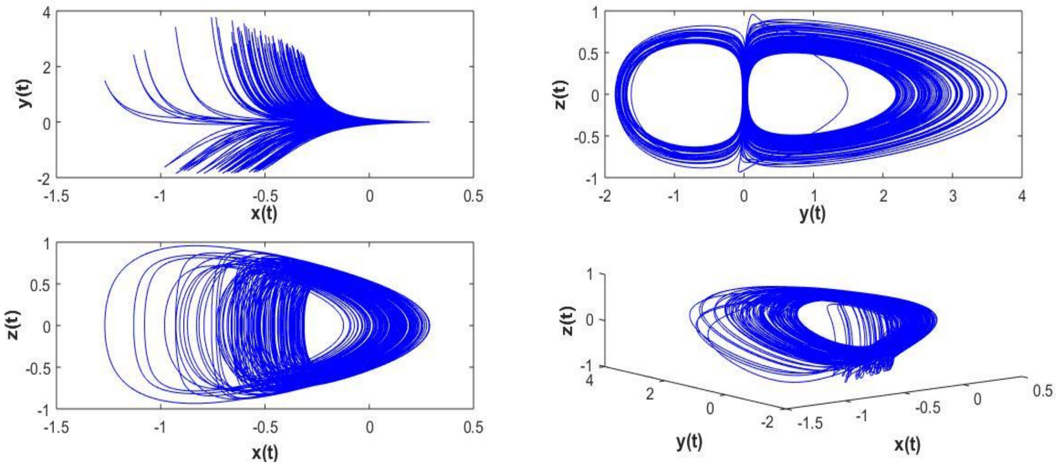

Figure.1. Phase portraits of the chaotic system (1)

Theorem 2.1. For $\alpha = 8$ and $\beta = 3$, all equilibria of the system (1) are unstable.

Proof.

The Jacobian matrix of the system (1) is defined as

$$J = \begin{pmatrix} 0 & 0 & 1 \\ -z^2 & -\alpha z - 2\beta yz & -\alpha y - \beta y^2 - 2xz \\ 4x^3 - 0.2x^2 y & -0.2x^2 y + 2y & 0 \end{pmatrix}$$

For equilibrium $E_1 = (0.8408964i, 0, 0)$, the system (1) is linearized and the Jacobian matrix is defined as

$$J(E_1) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -2.3784i & 0 & 0 \end{pmatrix}.$$

To gain its Eigen values, we let $|\lambda I - J(E_1)| = 0$ and their corresponding Eigen values are $\lambda_1 = 1.0905 - 1.0905i$, $\lambda_2 = -1.0905 + 1.0905i$ and $\lambda_3 = 0$.

For equilibrium $E_2 = (-0.8408964i,\ 0, 0)$, the system (1) is linearized and the Jacobian matrix is constructed as

$$J(E_2) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 2.3784i & 0 & 0 \end{pmatrix}$$ and their corresponding Eigen values are $\lambda_1 = 1.0905 + 1.0905i$, $\lambda_2 = -1.0905 - 1.0905i$, and $\lambda_3 = 0$.

For equilibrium $E_3 = (0.8408964,\ 0, 0)$, the system (1) is linearized and the Jacobian matrix is defined as

$$J(E_3) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 2.3784 & 0 & 0 \end{pmatrix}$$ and their corresponding Eigen values are $\lambda_1 = 1.5422$, $\lambda_2 = -1.5422$ and $\lambda_3 = 0$.

For equilibrium $E_4 = (-0.8408964,\ 0, 0)$, the system (1) is linearized and the Jacobian matrix is defined as

$$J(E_4) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -2.3784 & 0 & 0 \end{pmatrix}$$ and their corresponding Eigen values are $\lambda_1 = 1.5422i$, $\lambda_2 = -1.5422i$ and $\lambda_3 = 0$.

Since the linearization matrices $J(E_1), J(E_2), J(E_3)$ and $J(E_4)$ have atleast one eigen values with positive real parts, then the equilibrium points E_1, E_2, E_3 and E_4 are unstable.

3. SYNCHRONIZATION OF THE PROPOSED CHAOTIC SYSTEM

In this section, we defined parallel systems of a chaotic dynamical system. These parallel systems of the chaotic system also show chaotic behavior. In [33], for $r = 2$ and $r = \frac{1}{2}$ the corresponding parallel systems of our chaotic system (1) are given by

Parallel System I:

$$\begin{aligned} \dot{x}_1 &= z_1 \\ \dot{y}_1 &= -z_1(\alpha y_1 + \beta y_1^2 + 2x_1 z_1) \\ \dot{z}_1 &= 2x_1^4 - 0.2x_1^2 y_1^2 + 2y_1^2 - 0.5 \end{aligned} \tag{3}$$

Parallel System II:

$$\begin{aligned} \dot{x}_2 &= z_2 \\ \dot{y}_2 &= -z_2(\alpha y_2 + \beta y_2^2 + \frac{1}{2} x_2 z_2) \\ \dot{z}_2 &= \frac{1}{2} x_2^4 - \frac{0.1}{2} x_2^2 y_2^2 + \frac{1}{2} y_2^2 - 0.5 \end{aligned} \tag{4}$$

Different Phase portraits of the chaotic system (1) and its parallel systems (3) and (4) are depicted in Figure.2.

3.1. Dislocated Phase Synchronization

Consider the proposed chaotic system (1) as the master system and its parallel system (3) with controllers as the slave system.

$$\begin{aligned} \dot{x}_1 &= z_1 + u_1 \\ \dot{y}_1 &= -z_1(\alpha y_1 + \beta y_1^2 + 2x_1 z_1) + u_2 \\ \dot{z}_1 &= 2x_1^4 - 0.2x_1^2 y_1^2 + 2y_1^2 - 0.5 + u_3 \end{aligned} \tag{5}$$

where u_1, u_2 and u_3 are controllers to be designed.

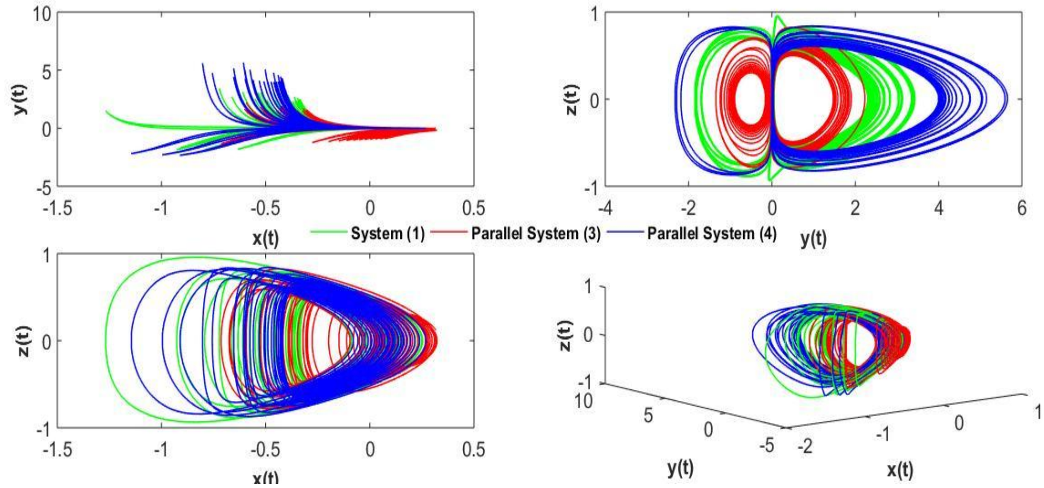

Figure.2. Phase portraits of the chaotic system (1) and its parallel systems (3) and (4)

Define the dislocated phase synchronization error as

$$\begin{aligned} e_1 &= x - z_1 \\ e_2 &= y - x_1 \\ e_3 &= z - y_1 \end{aligned} \tag{6}$$

The derivative of the error system (6) is

$$\begin{aligned} \dot{e}_1 &= \dot{x} - \dot{z}_1 \\ \dot{e}_2 &= \dot{y} - \dot{x}_1 \\ \dot{e}_3 &= \dot{z} - \dot{y}_1 \end{aligned} \tag{7}$$

Substituting (1) and (5) in (7), one can obtained that

$$\begin{aligned} \dot{e}_1 &= z - 2x_1^4 + 0.2x_1^2y_1^2 - 2y_1^2 - 0.5 - u_3 \\ \dot{e}_2 &= -z(\alpha y + \beta y^2 + xz) - z_1 - u_1 \\ \dot{e}_3 &= x^4 - 0.1x^2y^2 + y^2 - 0.5 + z_1(\alpha y_1 + \beta y_1^2 + 2x_1z_1) - u_2 \end{aligned} \tag{8}$$

Theorem 3.1. Consider the master system (1) and slave system (5). If the feedback gain controller $k_i > 0, i = 1,2,3$ then the state trajectories of an error dynamical system (8) converge to 0 for the total control laws

$$\begin{aligned} u_1 &= -z(\alpha y + \beta y^2 + xz) - z_1 + k_1e_1 \\ u_2 &= x^4 - 0.1x^2y^2 + y^2 - 0.5 + z_1(\alpha y_1 + \beta y_1^2 + 2x_1z_1) + k_2e_2 \\ u_3 &= z - 2x_1^4 + 0.2x_1^2y_1^2 - 2y_1^2 + 0.5 + k_3e_3 \end{aligned} \tag{9}$$

In other words, the dislocated phase synchronization between systems (1) and (5) is achieved for the total control laws (9).

Proof.

Substituting (9) in (8), we get

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \tag{10}$$

Consider the Lyapunov candidate function as

$$V\left(e(t)\right) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{11}$$

Then the time derivative of $V\left(e(t)\right)$ can be written as

$$\begin{aligned} \dot{V}\left(e(t)\right) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ &= -[k_1 e_1^2 + k_2 e_2^2 + k_3 e_3^2] \\ \therefore\ \dot{V}\left(e(t)\right) &= -e^T P_1 e \end{aligned} \tag{12}$$

where $P_1 = \begin{matrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{matrix}$

Then the error system (8) is asymptotically stable if the matrix P_1 should be positive definite.

The necessary and sufficient conditions for a matrix P_1 to be positive definite if the diagonal elements of P_1 must be all positive and the determinants of all the upper left-hand corners of P_1 are positive.

Hence P_1 is positive definite, then $\dot{V}\left(e(t)\right) < 0$ is negative definite. Which implies the error system (8) is asymptotically stable. Based on Lyapunov stability theory, $\lim_{t} e_i(t) = 0, i = 1,2,3$. Therefore the systems (1) and (5) are synchronized successfully.

3.2. Numerical Simulation

If $k_1 = 15,\ k_2 = 5$ and $k_3 = 3,$ then the synchronized error system (10) becomes

$$\begin{aligned} \dot{e}_1 &= -15 e_1 \\ \dot{e}_2 &= -5 e_2 \\ \dot{e}_3 &= -3 e_3 \end{aligned} \tag{13}$$

The time variation of the error system (13) using dislocated phase synchronization is depicted in Figure.3. It shows that, systems (1) and (5) are completely synchronized after a time $t > 1.5$.

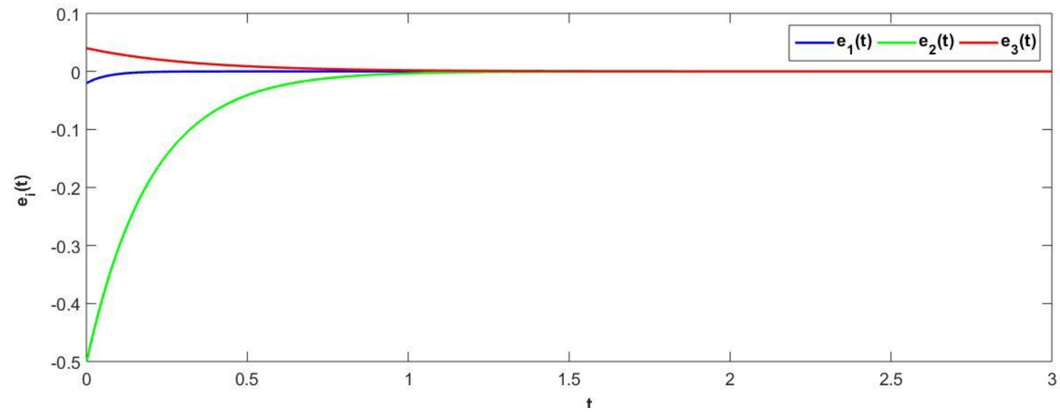

Figure.3. The dislocated phase synchronized error trajectories of parallel systems (1) and (5)

3.3. Dislocated Anti-Phase Synchronization

Consider the proposed chaotic system (1) as the master system and its parallel system (4) with controllers as the slave system.
The controlled system is

$$\begin{aligned} \dot{x}_2 &= z_2 + v_1 \\ \dot{y}_2 &= -z_2(\alpha y_2 + \beta y_2^2 + \frac{1}{2}x_2 z_2) + v_2 \\ \dot{z}_2 &= \frac{1}{2}x_2^4 - \frac{0.1}{2}x_2^2 y_2^2 + \frac{1}{2}y_2^2 - 0.5 + v_3 \end{aligned} \tag{14}$$

where v_1, v_2 and v_3 are controllers to be designed.
The dislocated anti-phase synchronization error system is defined as

$$\begin{aligned} E_1 &= x + z_2 \\ E_2 &= y + x_2 \\ E_3 &= z + y_2 \end{aligned} \tag{15}$$

The derivative of error system (15) is

$$\begin{aligned} \dot{E}_1 &= \dot{x} + \dot{z}_2 \\ \dot{E}_2 &= \dot{y} + \dot{x}_2 \\ \dot{E}_3 &= \dot{z} + \dot{y}_2 \end{aligned} \tag{16}$$

Substituting (1) and (14) in (16), one can get

$$\begin{aligned} \dot{E}_1 &= z + \frac{1}{2}x_2^4 - \frac{0.1}{2}x_2^2 y_2^2 + \frac{1}{2}y_2^2 - 0.5 + v_3 \\ \dot{E}_2 &= -z(\alpha y + \beta y^2 + xz) + z_2 + v_1 \\ \dot{E}_3 &= x^4 - 0.1x^2 y^2 + y^2 - 0.5 - z_2(\alpha y_2 + \beta y_2^2 + \frac{1}{2}x_2 z_2) + v_2 \end{aligned} \tag{17}$$

Theorem 3.2. The dislocated anti-phase synchronization between systems (1) and (14) is achieved for the following nonlinear control laws:

$$
\begin{aligned}
v_1 &= z(\alpha y+\beta y^2+xz)-z_2-K_1E_1\\
v_2 &= -x^4+0.1x^2y^2-y^2+0.5+z_2(\alpha y_2+\beta y_2^2+\frac{1}{2}x_2z_2)-K_2E_2\\
v_3 &= -z-\frac{1}{2}x_2^4+\frac{0.1}{2}x_2^2y_2^2-\frac{1}{2}y_2^2+0.5-K_3E_3
\end{aligned}
\tag{18}
$$

where K_1, K_2 and K_3 are positive feedback gains which will be evaluated in order to achieve synchronization.

Proof:

Substituting (18) in (17), one can get

$$
\begin{aligned}
\dot{E}_1 &= -K_1E_1\\
\dot{E}_2 &= -K_2E_2\\
\dot{E}_3 &= -K_3E_3
\end{aligned}
\tag{19}
$$

Consider the Lyapunov candidate function as

$$
V\left(E(t)\right)=\frac{1}{2}\left(E_1^2+E_2^2+E_3^2\right) \tag{20}
$$

Then the time derivative of $V\left(E(t)\right)$ can be written as

$$
\begin{aligned}
\dot{V}\left(E(t)\right) &= E_1\dot{E}_1+E_2\dot{E}_2+E_3\dot{E}_3\\
&= -K_1E_1^2-K_2E_2^2-K_3E_3^2\\
&= -[K_1E_1^2+K_2E_2^2+K_3E_3^2]\\
\dot{V}\left(E(t)\right) &= -E^TP_2E
\end{aligned}
\tag{21}
$$

where $P_2 = \begin{matrix} K_1 & 0 & 0\\ 0 & K_2 & 0\\ 0 & 0 & K_3\end{matrix}$ is a positive definite matrix. It shows that $\dot{V}\left(E(t)\right)<0$ for all $K_i>0$.

According to Lyapunov stability theory, $\lim_{t} E_i(t)=0,\ i=1,2,3.$ Hence the synchronization between systems (1) and (14) is achieved.

3.4. Numerical Simulation

If $K_1=25,\ \mathrm{K}_2=10$ and $K_3=6,$ then the synchronized error (10) becomes

$$
\begin{aligned}
\dot{E}_1 &= -25E_1\\
\dot{E}_2 &= -10E_2\\
\dot{E}_3 &= -6E_3
\end{aligned}
\tag{22}
$$

The time variation of the error system (22) is depicted in Figure.4. It shows that, the dislocated anti-phase synchronization errors between the systems (1) and (14) are stable after a time $t>3$.

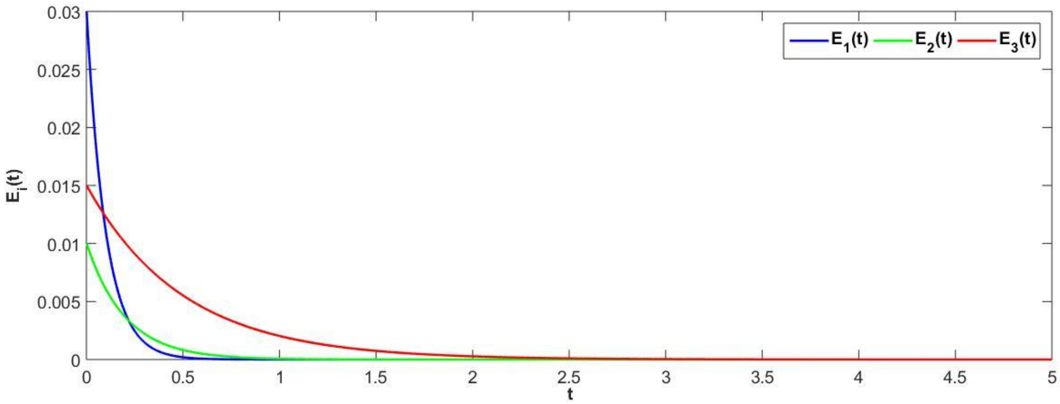

Figure.4. The dislocated phase synchronized error trajectories of parallel systems (1) and (14)

4. APPLICATION OF THE PROPOSED CHAOTIC SYSTEM

Encryption is a way to secure and verify data that are traded through public communication channels in the presence of intruder party called antagonists. Consequently, the transmitted or stored message can be converted to unreadable form except for intended receivers. The decryption techniques allows intended receiver to reveal the contents of previously encrypted message via secrete keys exchanged exclusively between transmitter and receiver.

The encryption and decryption techniques can be applied equally to a message in any form such as text, image, audio or video.

In this section, soundtrack encryption - decryption algorithm is constructed based on synchronized chaotic systems are applied to real-time sound track encryption and decryption using the following proposed algorithm:

Encryption algorithm:

1. Let S_T be an original soundtrack.
2. Consider the master system (1) and the slave system (3) as Sender's $(\mathcal{S})$ and Receiver's $(\mathcal{R})$ systems.
3. $\mathcal{S}$ and $\mathcal{R}$ agree on N and $t \geq t_0$, where $t_0 = 2$.
4. $\mathcal{S}$ computes the solutions $x(t), y(t)$ and $z(t)$ from the system (1) at time t and generate his/her own secret key $\mathcal{S}_k$ is as follows: $\mathcal{S}_k = \lfloor x(t) + y(t) + z(t) \times 10^4 \rfloor \ mod\ N$
5. $\mathcal{R}$ computes the solutions $x_1(t), y_1(t)$ and $z_1(t)$ from the system (3) at time t and generate his/her own secret key $\mathcal{R}_k$ is as follows: $\mathcal{R}_k = \lfloor x_1(t) + y_1(t) + z_1(t) \times 10^4 \rfloor \ mod\ N$
6. $\mathcal{S}$ wants to share a soundtrack S_T to $\mathcal{R}$. Then $\mathcal{S}$ computes $\mathrm{E} = S_T * \mathcal{S}_k \,(mod\ N)$ and send it to $\mathcal{R}$.

Decryption algorithm:

7. $\mathcal{R}$ receives an encrypted soundtrack E and receives original soundtrack $\mathcal{D} = \mathrm{E} - \mathcal{R}_k^{-1}\,(mod\ N)$

$$\mathcal{D} = S_T * [\mathcal{S}_k * \mathcal{R}_k^{-1}(mod\ N)]$$
$$= S_T * \ x(t) + y(t) + z(t) \times 10^4 \ * \ x_1(t) + y_1(t) + z_1(t) \times 10^4 \ ^{-1}(mod\ N)$$
$$\mathcal{D} = S_T \ (\because x(t) = z_1(t),\ y(t) = x_1(t) \text{ and } z(t) = y_1(t) \text{ after some time } t \geq t_0)$$

4.1 Numerical Simulation

Consider the given soundtrack wave file S_T . Assume that $\mathcal{S}$ and $\mathcal{R}$ agree on $N = 250$ and $t = 12$ (Synchronization error between the system (1) and (5) are tends to zero after a time $t_0 = 2$). The sender computed the solutions $x(t) = -0.4936, y(t) = 2.463$ and $z(t) = -0.0075$ at t and generate his/her own secrete key $\mathcal{S}_k \equiv 69\,(mod\ 250)$. $\mathcal{R}$ computes the solutions $x_1(t) = -0.4936,\ y(t) = 2.463$ and $z_1(t) = -0.0075$ at $t = 12$ and generate his/her own secrete key $\mathcal{R}_k \equiv 69\,(mod\,250)$. $\mathcal{S}$ wants to share soundtrack S_T and it is depicted in Figure.5.

For encryption, $\mathcal{S}$ computes the encrypted sound track $\mathrm{E} = S_T * \mathcal{S}_k\,(mod\ N)$ and send it to $\mathcal{R}$. $\mathcal{R}$ receives an encrypted soundtrack E which is depicted in Figure. 6.

For decryption, $\mathcal{R}$ recovers an original soundtrack by computing

$$\mathcal{D} = \mathrm{E} * (\mathcal{R}_k)^{-1}\,(mod\ N)$$
$$= S_T * \left(69 * (69)^{-1}\right)(mod\ N)$$
$$\mathcal{D} = S_T$$

The decrypted soundtrack is shown in Figure.7.

Remark

The decrypted soundtrack has no information loss in proposed algorithm. Since error between the original and decrypted soundtrack is zero, which is shown in Figure.8.

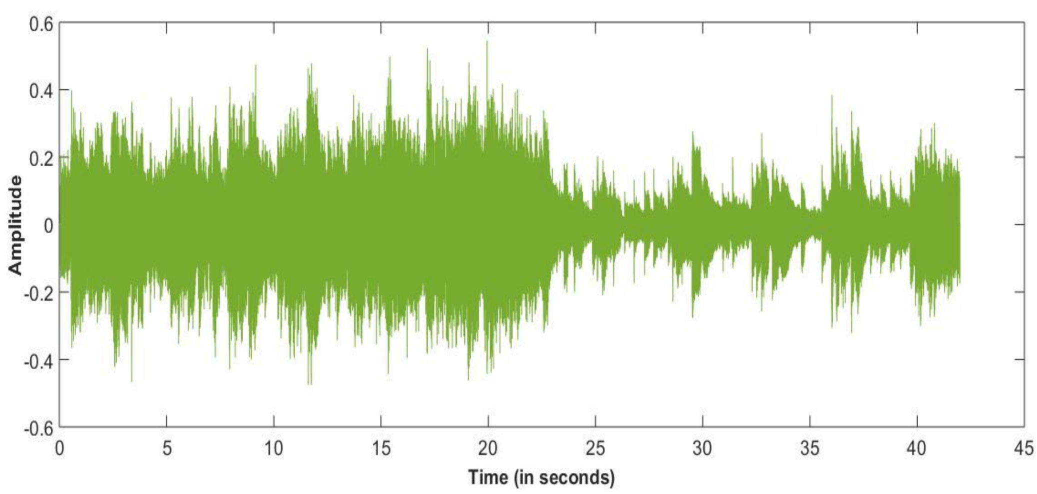

Figure.5. Original soundtrack.

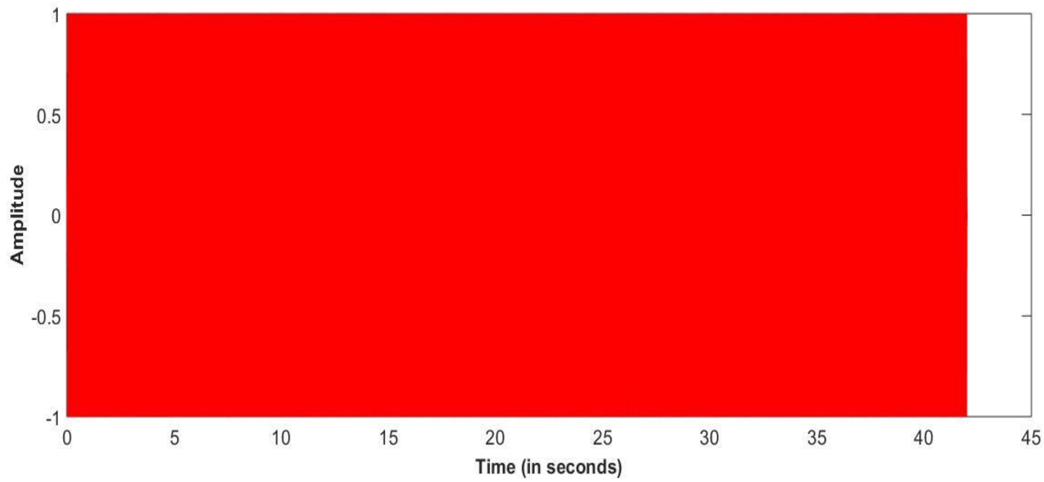

Figure.6. Encrypted soundtrack.

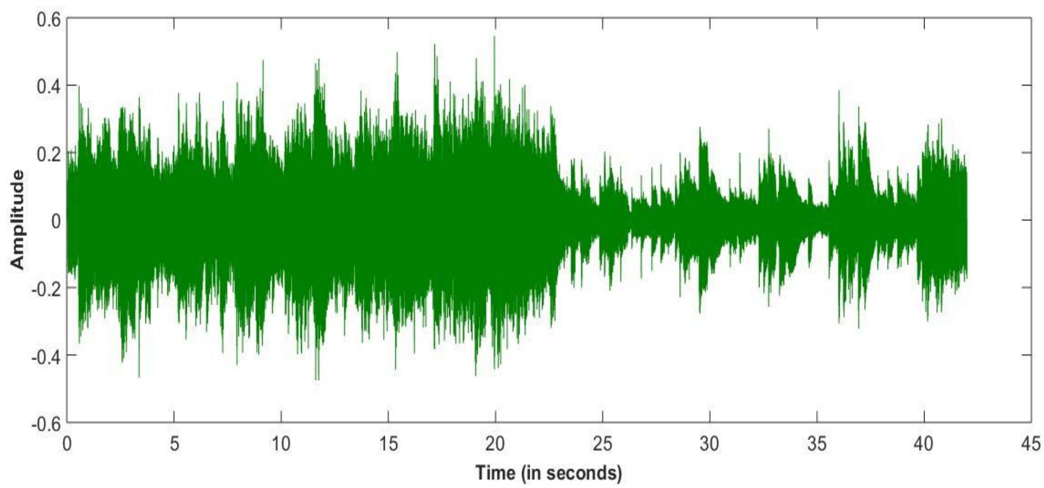

Figure.7. Decrypted soundtrack.

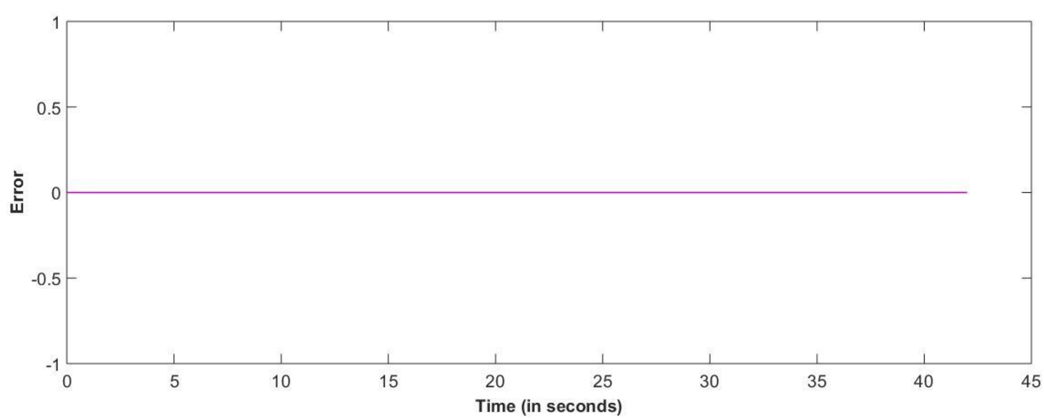

Figure.8. Error between encrypted and decrypted soundtrack

5. CONCLUSIONS

This work looked into the dissipative, equilibrium, and stability properties of a three-dimensional integer-order chaotic system. A appropriate controller was developed to enable dislocated phase and anti-phase synchronization of two parallel chaotic systems. A new technique for soundtrack encryption and decryption was developed using the proposed dislocation synchronization mechanism. Numerical simulations were provided to show how well the theoretical results worked.

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