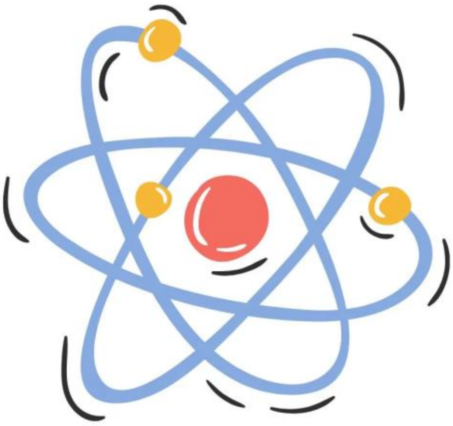

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THE DYNAMICAL ORIGIN OF THE MASSIVE GRAVITON AND DECOUPLING OF TORSION

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ABSTRACT: When a direction vector is attached to a space-time point, we may really express it as a spin. opposite orientations of the vector produce opposite helicities, which indicate states of spin up and down. Discreteness embedded in the space-time manifold M_4 gives rise to a lattice where Lorentz symmetry is broken due to the noncommutativity of space-time coordinates, resulting in an anisotropic space. But when the noncommutativity parameter falls to the usual symplectic matrix, space-time and torsion are no longer connected, and general relativity realizes gravity with two additional degrees of freedom: a scalar field and a heavy graviton. The condition is similar to spin-charge separation of Yang-Mills gauge fields in the low energy domain.

KEYWORDS:

1 Introduction

Here, we will demonstrate that the manifold $M_4 \times Z_2$ relates to a twisted space-time in the context of non-commutative space-time. More precisely, it has been demonstrated that the corresponding space-time leads to teleparallel gravity when the non-commutativity parameter $\theta^{\mu\nu}$ has a functional dependence on phase space variables, but decouples from the contorted space-time and gives rise to a massive graviton and scalar field when $\theta^{\mu\nu}$ reduces to a constant matrix torsion, as observed by Chamseddine [1]. The spin-charge duality connection is reproduced here in the duality relation between torsion and curvature, and the situation is similar to the spin-charge separation in the low energy domain found in $SU(2)$ Yang-Mills theory [2]. When the non-vanishing ground state expectation value of this field gives rise to the mass of the gauge boson, the additional degrees of freedom involving the massive graviton and a scalar field effectively correspond to the appearance of a scalar field in spin-charge separation of $SU(2)$ Yang-Mills theory. Similar to how spin and charge recombine in the high energy (short-distance) region to produce the asymptotically free Yang-Mills gauge theory, in the $SL(2,C)$ gauge theoretical formulation of gravity, the functional dependence of the noncommutativity parameter $\theta^{\mu\nu}$ results in the twisted space-time that gives rise to teleparallel gravity. We will talk about the creation of enormous gravitons and the separation of torsion from twisted space-time when the noncommutativity parameter manifests as a constant matrix.

2. Theoretical Background

Torsion and Massive Graviton Decoupling

We have argued that when the non-commutativity parameter $\theta^{\mu\nu}$ is provided by the field strength $F_{\mu\nu}$, the gauge theoretical extension of space-time coordinates leads to the noncommutative geometry.

Evidently we note that this will give rise to commutation relations

$$[Q_\mu, Q_\nu] = [F_{\mu\nu}(p)]$$

$$\Omega = \left(\frac{1}{2}\right) g^{ij} dp_i \Lambda dq_j \qquad (1)$$

where $F_{\mu\nu}$ is the field strength of the non-Abelian gauge field

According to equation (1), this is a function of the momentum variable $F_{\mu\nu}(p)$ in coordinate space, while the field strength is a function of the space variable $F_{\mu\nu}(p)$ in momentum space due to non-commutativity. We note that the topological current is $\vec{J}^{\mu}_{\theta}$ when there is this functional dependency.

$$\vec{J}^{\mu}_{\theta} = \left(\varepsilon^{\mu\nu\alpha\beta} \vec{A}_\nu \times \vec{F}_{\alpha\beta}\right) = \left(\varepsilon^{\mu\nu\alpha\beta} \partial_\nu \vec{F}_{\alpha\beta}\right) \qquad (2)$$

is non-vanishing. Space-time becomes twisted as a result of the non-commutative manifold's torsion caused by this current. In contrast, when we have the relation $\partial_\mu F_{\mu\nu} = 0$, we discover that $F_{\mu\nu}$ is a constant matrix, meaning that the non-commutativity parameter $\theta^{\mu\nu}$ which is crucial in the star product formulation of the non-commutativity of space—is identical with the typical symplectic matrix $\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$ after normalization. Torsion is disconnected from space-time in this scenario, presumably due to diminishing current and the twisted space-time. Clearly, general relativity is used to produce gravity in this situation. However, the impact of torsion will be realized through some extra degrees of freedom in general relativity because of the implicit structure of twisted space-time in this non-commutative manifold. Indeed, any component of J^{μ}_{θ} may be decomposed as

$$\vec{J}_{\theta}^{\mu}=\left(\varepsilon^{\mu\nu\lambda\sigma}\partial_{\nu}\vec{F}_{\lambda\sigma}\right)=\vec{n}^{\mu}\varphi(x) \qquad (3)$$

where $\vec{n}^{\mu}$ is the unit axial vector. Thus, we note that the decoupling of torsion will give rise to a scalar $\varphi(x)$. Besides, the axial vector nature of this topological current $\vec{J}_{\theta}^{\mu}$ which is related to chiral anomaly gives rise to the relation for any component of the current [3]

$$\Box J_{\theta}^{\mu}=\left(\tilde{m}^{2}J_{\theta}^{\mu}\right) \qquad (4)$$

where $\tilde{m}$ corresponds to a constant having the dimension of mass. This leads to the relation

$$\Box\,{}^{*}F_{\mu\nu}=\left(\tilde{m}^{2}\,{}^{*}F_{\mu\nu}\right) \qquad (5)$$

suggesting that the gauge field is now massive. This corresponds to the realization of a massive graviton when gravitation is achieved through general relativity. Thus, we note that when the noncommutativity parameter $\theta^{\mu\nu}$ is a constant matrix, we will have additional degrees of freedom in general relativity given by a massive graviton and a scalar field. This result is identical with that obtained by Chamseddine [1] in the $SL(2,C)$ gauge theory of gravity in a noncommutative space with constant $\theta^{\mu\nu}$.

It should be noted that the spin-charge separation seen in Yang-Mills theory in the weak coupling limit [4–7] is comparable to the decomposition equation (3). Yes, we are able to write for any J_{θ}^{μ} component.

$$J_{\theta}^{\mu}=\left(\varepsilon^{\mu\nu\lambda\sigma}\partial_{\nu}F_{\lambda\sigma}\right)=n^{\mu}\varphi(x) \qquad (6)$$

where n^{μ} is a component of the unit axial vector. This suggests that while the internal index, or "charge," is contained in $\varphi(x)$, the gauge field's spin is encoded in n^{μ}. Similar to the separation of spin charges in Yang-Mills $SU(2)$ theory, where a scalar field is involved and its vacuum expectation value determines the gauge boson mass, in this instance, we have noted that a scalar field and a massive graviton are involved in the decoupling of torsion from space-time. Furthermore, just as we have a duality relation between spin and charge in the instance of spin-charge separation in the $SU(2)$ gauge field, we also have a duality relation between torsion and curvature in this situation of torsion decoupling. In fact, general relativity's counterpart of teleparallel gravity should be used in twisted space-time when torsion is decoupled. On the other hand, the large graviton plus a scalar field will cause two more degrees of freedom due to the implicit torsion. It can be seen that the twisted space-time will reduce to Riemann-Cartan space-time U_4 in the context of commutative Riemannian space, where curvature and torsion manifest as separate degrees of freedom. The teleparallel gravity general relativity equivalency in this commutative instance will not introduce an extra degree of freedom.

It should be noted that teleparallel gravity implies the implied presence of a monopole since it stems from the functional dependency of the noncommutativity parameter $F_{\mu\nu}(\phi)$, where ϕ is a phase space variable $p(q)$ violating the associativity rule [5-8]. Thus, in a non-Abelian gauge theory, there exists a concealed Abelian gauge field.

This is associated with the topological Lagrangian is given by

$$L = \left(\frac{1}{4}\right) Tr(\varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}) \qquad (7)$$

eqn. (7) which gives rise to torsion.

In fact, the inclusion of this Lagrangian in a non-Abelian gauge theory shows that the gauge orbit space is multiply-connected and that a vortex line exists, which is topologically comparable to a magnetic flux line [9–12]. Therefore, it is possible to see the teleparallel equivalency of general relativity as an expression of the non-Abelian $SL(2,C)$ gauge theory's hidden Abelian gauge field.

3. Conclusion

When the noncommutativity parameter $\theta^{\mu\nu}$ is a constant matrix, we find that we have decoupling of torsion from twisted space-time, giving rise to the massive graviton and a scalar field. This is analogous to the spin-charge separation in the weak coupling limit of the Yang-Mills gauge field, which is coupled to a scalar field and yields massive gauge bosons from its vacuum expectation value. As in the case of spin-charge separation, the duality between spin and charge degrees of freedom is also present here. Curvature and torsion also have a dualistic relationship. Since we have a Riemannian structure in the zero torsion limit that corresponds to Einstein's general relativity, a teleparallel theory of gravity may be understood as the zero curvature reduction of the Einstein-Cartan space-time, which is characterized by both nonvanishing torsion and curvature. Curvature and torsion also have a dualistic relationship. Since we have a Riemannian structure in the zero torsion limit that corresponds to Einstein's general relativity, a teleparallel theory of gravity may be understood as the zero curvature reduction of the Einstein-Cartan space-time, which is characterized by both nonvanishing torsion and curvature.One may note that teleparallel gravity and general relativity are equivalent under the Weitzenbock geometry, where the curvature of the Einstein-Cartan manifold U_4 is assumed to be vanishing. There are various notable aspects to this geometry. It leads to a pure tensorial demonstration of energy positivity in general relativity, as demonstrated by Nester [13–22]. It produces a natural introduction of Ashtekar variables, as noted by Mielke [23].

4. Discussion

Finally, we may note that the noncommutative mani-fold $M_4 \times Z_2$ leads to a scalar field with a massive graviton, just as $\theta^{\mu\nu}$ is a constant matrix; this is analogous to the electroweak theory, where the gauge bosons get mass. In light of this, it is found that electroweak theory and gravity are comparable. thus noncommutative manifold, $M_4 \times Z_2$, is really associated with the quantization of a fermion in flat space, and thus makes it feasible to understand the connection between quantum physics and gravity through it. Consequently, noncommutative geometry provides a means of reconciling other forces of particle physics with general relativity.

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