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ABSTRACT: It is possible to see that this geometric formalism broadens the situation in this instance. Indeed, the Pontryagin term and the Berry phase are related to chiral anomaly; this may be viewed as an extension of the Bohm-Ahranov effect. In this sense, a background magnetic field is effectively attached by inserting a direction vector or vortex line connected with a space-time point, and magnetic charge is effectively represented by the charge corresponding to the gauge field. Thus, there may be a connection between the geometry of a vortex line and that of a charged particle passing through the magnetic monopole's field. There is a hole in the gauge orbit space U/G in the 3+1 dimension since it has a ring-like structure. As a result, there is some magnetic flux passing through the gauge orbit space hole. Given this, it is reasonable to assume that the Bohm-Ahranov kind of effect in ordinary space is the source of the vacuum. Thus, it may be assumed that the same geometrical characteristic gives birth to the topology of the gauge orbit space in in 2+1 dimension, which corresponds to the topology of a sphere representing a magnetic monopole. We may take the disconnected gauge group into consideration, which would produce a non-Abelian gauge structure and render the theory asymptotically free.

KEYWORDS: Pontryagin Term, Berry Phase, Chiral Anomaly, Magnetic Monopole, Non-Abelian Gauge Structure

1. INTRODUCTION

The primary geometrical feature underlying the topological term in the non-Abelian gauge field Lagrangian is found to be an intrinsic anisotropic feature that causes the quantization of a Fermion [1] in an 3+ 1 dimension. This anisotropy is brought about by attaching a "direction vector" or "vortex line" to a space-time point. A particle moving in such an anisotropic space moves similarly to a charged particle moving in front of a magnetic monopole. An additional benefit of this formalism is that it eliminates chiral anomaly in theory when fermions are considered in chiral forms. Additionally, it proposes the creation of a disconnected gauge group for chiral fermions and advances asymptotically free quantum electrodynamics involving chiral fermions. In Sec. I, we will cover various aspects related to the gauge orbit space, while in Sec. II we will cover asymptotic freedom and some characteristics of a non-Abelian gauge field that emerges from a disconnected Abelian gauge group.

2. THEORETICAL BACKGROUND

I. Gauge field in Abelian theory inside a non-Abelian framework with space gauge orbit

As seen above, a non-Abelian gauge theory with a Pontryagin term in the 3+1 dimension and a Chem-Simons term in the 2+1 dimension has a concealed Abelian gauge field. The field theory under examination can thus be thought of, as suggested by Wu and Zee [2], as a particle traveling in the space U of non-Abelian gauge potentials

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under the influence of an Abelian electromagnetic potential A, whose position is represented by A (non-Abelian gauge potential). In the language of differential forms, we can write,

$$A = (g^{-1}\partial g) + (g^{-1}ag) \qquad (1)$$

where

$$A = (A_i \partial x^i) \ , \ a = (a_i \partial x^i)$$

Where G indicates the space of local gauge transformation $g(x)$, which is made up of the points $a(x)$, is the space of gauge orbits U/G .

Following Wu and Zee [2] we can consider a homotopic analysis of this gauge-orbit space. Recalling that $[\pi_3(g)] = Z$ for all simple non-Abelian groups G and $\pi_2(G) = 0$, $\pi_n(U) = 0$ for all n, this helps to write

$$[\pi_n(U/G)] = \pi_{n-1}(G) \quad \text{for} \quad n\rangle 1 \qquad (2)$$

In 3+ 1 dimension

$$\pi_1(U/G) = \pi_0(G) = \pi_3(G) = (Z) \qquad (3)$$

The requirement that the gauge transformation $g(x)$ approaches a constant regardless of the direction of x as $x \to \alpha$ leads to the equality $\pi_0(G) = \pi_3(G)$. As a result, U/G is multiple linked, has a ring-like topology, and a vortex line-corresponding field strength.

$$\pi_1(U/G) = \pi_0(G) = \pi_2(G) = 0 \qquad (4)$$

Considering the second homotopy instead

$$\pi_2(U/G) = \pi_1(G) = \pi_3(G) = (Z) \qquad (5)$$

This implies that the topology of U/G is akin to a sphere, and the appropriate magnetic monopole-like field intensity is found.

From the quantum geometry as developed here we observed that in 3+1 dimension the Pontryagin term arises out of SL(2,c) invariance in spinor affine spade where the gauge fields belong to SL(2,c). These SL(2,c) gauge fields may be traced back to be generated from the spinorial variables in the metric $g_{\mu\nu}(x,\theta,\bar{\theta})$ and corresponds to the direction vector (vortex line) attached to the space-time point.

It has been demonstrated that this results in the Pontryagin index

$$q = \int J_0^2 (d^3 x) = \int_{surface} (\in^{ijk} \, d\sigma_i \, f_{jk}^2) \qquad (i, j, k = 1, 2, 3)$$

which suggests f_{jk}^2 to be magnetic field like components for the background vector potential $b_i^{(2)}$, q is actually assoriated with the magnetic pole strength.

Essentially, a background magnetic field may be associated with a direction vector or vortex line related to a space-time point, and magnetic charge can be represented by the charge corresponding to the gauge field. A charged particle traveling in the magnetic monopole's field may thus share some geometry with a vortex line. Given that the gauge orbit space U/G in the 3+1 dimension has a ring topology, this suggests that the space contains a hole. As a result, there is some magnetic flux passing through the gauge orbit space hole. In view of this the vacuum may be taken to arise from the Bohm-Ahranov type of effect in ordinary space. In 2+1 dimension the topology of the gauge orbit space corresponding that of a sphere representing a magnetic monopole may thus be taken to arise from the same geometrical feature.

Here, we may note that this geometrical formalism makes the situation more broad. In fact, the relationship with chiral anomaly links the Pontryagin term to the Berry phase [3-5], and this Berry phase may be thought of as a more extended form of the Bohm-Ahranov effect. In fact, it has been shown that the Berry phase is given by the relation $e^{i\phi_\beta} = e^{i2\pi\mu}$ where

$$2\mu = q = \int J_\nu^2 d^3 x = \int (\partial_\mu J_\mu^2 d^4 x) = -\frac{1}{2}\int (\partial_\mu J_\mu^5 d^4 x) \qquad (6)$$

Thus the background Abelian gauge field when the θ-tern is introduced in a non-Abelian gauge theory is effectively responsible for the Berry phase.

II . Superspace QED and Deterministic Freedom

A disconnected gauge group must be introduced for the external Abelian field that interacts chirally symmetrically with the matter field. This is because the chiral description of the fermions in 3+1 dimension in terms of the spinorial variables , in the metric tensor gives rise to SL(2,c) gauge field current, as noted in a recent paper [6–8].

In case, the external Abelian gauge field is the electromagnetic field (photon). the Lagrangian density is given by

$$L = -(\bar{\psi}\gamma_\mu D_\mu \psi) - \frac{1}{4}(\in^{\alpha\beta\gamma\delta} \, \tilde{F}_{\alpha\beta}\tilde{F}_{\gamma\delta}) - \frac{1}{4}Tr(F_{\mu\nu}F^{\mu\nu}) + Tr(J_\mu A^\mu) \qquad (7)$$

Here, D_μ is the SL(2,c) gauge covariant derivative and considering the order of $\psi - B_\mu$, coupling to be negligible compared to matter current electromagnetic field coupling we can replace it by ∂_μ .

We have

$$L = -(\bar{\psi}\gamma_\mu D_\mu \psi) - \frac{1}{4}(\in^{\alpha\beta\gamma\delta}\ \tilde{F}_{\alpha\beta}\tilde{F}_{\gamma\delta}) - \frac{1}{4}Tr(F_{\mu\nu}F^{\mu\nu}) + Tr(J_\mu A^\mu)\ ,\ B_\alpha \in \left[SL(2e)\right]$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \qquad (8)$$

A_μ, being the electromagnetic gauge potential and J_μ is the matter current matrix given by

$$J_\mu = \begin{bmatrix} (\bar{\psi}_R\gamma_\mu\psi_R + J^2_\mu) & 0 \\ 0 & (\bar{\psi}_L\gamma_\mu\psi_L - J^2_\mu) \end{bmatrix} \qquad (9)$$

J^2_μ, being the second part of the gauge field current of SL (2,c) that was covered in the previous section. It is evident that this matrix structure of J_μ, exhibiting chiral form suggests that for A_μ, we should take the disconnected gauge group [5] $U_{1L} \otimes U_{1R} = U_1 \times \{1, d\}$ where 'd' is the orientation reversing operation. It appears that in this interaction, the current and field strength are gauge covariant rather than gauge invariant, with each changing sign under 'd'. This is comparable to non-Abelian theories in which field strengths and currents are merely gauge covariant, even in the presence of identity-related gauge transformations.

The internal symmetry group here is $O(2)$ which is given by the relation

$$O(2) = [SO(2) \times \{1, d\}] = [U_1 \times \{1, d\}] \qquad (10)$$

Indeed, we can take

$$A_\mu = \begin{bmatrix} \left(A_{\mu^+}\right) & 0 \\ 0 & \left(A_{\mu^-}\right) \end{bmatrix} \qquad (11)$$

According to KisKis [9–10], we can take into consideration a massive system of observers, each in charge of a little open patch of the connected space-time manifold M. For the sake of argument, let us assume that each frame in M has the same orientation. This suggests that positive charge may be precisely defined everywhere and that the region is only physically connected. This suggests that the Lagrangian can incorporate the gauge field (connection).

$$L^i = (L^i_g + L^i_M) \qquad (12)$$

where i identifies quantities associated with the region U_i, L_M^i Lim is the matter field Lagrangian and L_g^i is the kinetic energy term for the connection. The gauge symmetry of the L_g^i is given by

$$A \to \{g^{-1}(\partial + A)g\} \qquad (13)$$

with g a smooth map

$$g = U_i \to [O(2)] \qquad (14)$$

which may lie in either component of $[O(2)]$. An orientation-reversing transformation at each location may be expressed as

$$g = dg_0,\ g_0 = U_i \to [SO(2)],\ d = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (15)$$

This gives

$$a = \{g_0^{-1}(\partial - A)g_\nu\} \qquad (16)$$

It is clear that it combines orientation-preserving gauge rotation with charge conjugation. It appears that the interaction between the chiral currents and the gauge field is unconnected in this approach. In fact, composing

$$A_\mu = \begin{bmatrix} (A_{\mu+}) & 0 \\ 0 & (A_{\mu-}) \end{bmatrix}$$

we find the interaction term is given by

$$\begin{bmatrix} \{(\bar{\psi}_R\gamma_\mu\psi_R + J_\mu^2)A_{\mu+}\} & 0 \\ 0 & \{(\bar{\psi}_L\gamma_\mu\psi_L - J_\mu^2)A_{\mu-}\} \end{bmatrix} \qquad (17)$$

Apparently, the Lagrangian does not contain a term similar to $A_{\mu+}\ A_{\mu-}$.

It is now seen that $\bar{\psi}_R\gamma_\mu\psi_R + J_\mu^2$ or $\bar{\psi}_L\gamma_\mu\psi_L - J_\mu^2$ is the fundamental current in this disconnected gauge field formalism.

It is possible to adequately simulate the background current originating from the imaginary magnetic monopole by a fictional chiral current [11–12].

$$\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = [\partial_\mu J_\mu^5 = -2\partial_\mu J_\mu^2] \qquad (18)$$

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Thus, the second component of the gauge field current is used to indicate the chiral anomaly in this context. We see that the addition of modifies chiral currents and causes the anomaly to disappear.

Indeed from the relation (18)

$$\partial_\mu J_\mu^2 = (-\frac{1}{2}\partial_\mu J_\mu^5)$$

we have the general solution

$$-J_\mu^2 \square (J_\mu^5 + J_\mu^V) \square \overset{\square}{\psi}_L \gamma_\mu \tilde{\psi}_L \qquad (19)$$

Where $\tilde{\psi}_L$ is a left handed fictitious spinor. Similarly we can take

$$J_\mu^2 = \tilde{J}_\mu^2 \square (\overset{\square}{\psi}_R \gamma_\mu \tilde{\psi}_R)$$

This helps us to formulate $SU(2)$ doublets $\begin{pmatrix} \psi_L \\ \tilde{\psi}_L \end{pmatrix}$, $\begin{pmatrix} \psi_R \\ \tilde{\psi}_R \end{pmatrix}$ and we can associate the gauge field inter action in terms of the $SU(2)$ triplet

$$\begin{pmatrix} A_{\mu+} \\ A_{\mu 0} \\ A_{\mu-} \end{pmatrix}$$

with the following Lagrangians:
where

$$\{(\bar{\psi}_R \gamma_\mu \psi_R + \overset{\square}{\psi}_R \gamma_\mu \psi_R) A_{\mu+}\}$$

$$\frac{1}{2}\{(\bar{\psi}_R \gamma_\mu \psi_R - \bar{\psi}_L \gamma_\mu \psi_L + \overset{\square}{\psi}_R \gamma_\mu \overset{\square}{\psi}_R - \overset{\square}{\psi}_L \gamma_\mu \overset{\square}{\psi}_L) A_{\mu 0}\}$$

$$\{(\bar{\psi}_L \gamma_\mu \psi_L + \overset{\square}{\psi}_L \gamma_\mu \psi_L) A_{\mu-}\} \qquad (20)$$

Where

$$A_{\mu 0} = \{\frac{1}{\sqrt{2}}(A_{\mu+} + A_{\mu-})\}$$

The unconnected gauge fields' "effective non-Abelian nature" leads us to propose that the accompanying interactions are now asymptotically free [13–17].

It should be mentioned that the typical minimum electromagnetic interactions with the magnetic field of the background may be expressed as

$$L = (ie\bar{\psi}\gamma_\mu \psi A_\mu + ie\overset{\square}{\psi}\gamma_\mu \bar{\psi} A_\mu) \qquad (21)$$

Due to their similar shape to those found in the original Lagrangian [18], the counter terms created by the fictional spinor will cancel the divergences at every order in perturbation theory. This is not unexpected because we have previously demonstrated that this for-malism [19-24] eliminates the chiral anomaly, which is often thought to result from regularization of short-distance singularities.

3. CONCLUSION

This paper deals with the quantization of a Fermi field in dimension. It includes the insertion of a direction vector or vortex line connected to a space-time point, which is associated with an inherent anisotropic property. In such an anisotropic space, the motion of a charged particle is analogous to that of a charged particle in the magnetic monopole's field.

4. DISCUSSION

This formalism indicates that the theory becomes asymptotically free in the case where the fermion current, separated into chiral form, helps formulate electromagnetic interactions in disconnected form. Ultimately, the topological index generated by the quantization of fermions inside an anisotropic space and its form is analogous to the strength of a magnetic pole.

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