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ABSTRACT: In this study, we have tried to investigate the geometric interpretation of Wu and Zee results. We shall show how the same geometrical feature gives rise to topological terms in the non-Abelian gauge field Lagrangian in dimensions 3+ 1 and 2+1. This specific geometrical feature allows for the Berry phase in quantum physics, which is an elongated version of the Bohm-Ahranov phase. When quantization corresponds to freezing the particles at their initial Landau level, this geometrical approach to the phase space quantization may be understood in terms of a global magnetic field acting on a free particle in a higher dimension configuration space. This method subsequently reveals the crucial function of the gauge field, which results in the Klauder and stochastic quantization equivalencies. This article presents a geometrical formalism of non-Abelian gauge theory with a term, specifically focusing on the importance of Abelian gauge structures in non-Atrlian theories. The proof demonstrates that when fermionic currents are expressed in chiral forms.

KEYWORDS: Vortex Dynamics, Non-Abelian Gauge Field, Topological Terms, Berry Phase, Geometric Quantization

1. INTRODUCTION

Wu and Zee [1] have shown in a recent study that the configuration space of non-Abelian gauge theories contains certain nontrivial Abelian background gauge fields when topological Lagrangian is included. In particular the θ -term and the topological mass term leads, respectively, to a vortex and a monopole in gauge orbit space in 3+1 and 2+ 1 dimensions. In view of this the θ-vacuum may be considered to arise from a kind of Bohm-Ahranov effect. The nature of the Abelian gauge field in a 3+1 dimension is found to be the field of a vortex line and in a 2+1 dimension it is the field of a monopole. This specific geometrical feature is responsible for the realization of the Berry phase which is a generalized version of the Bohm-Ahranov phase in quantum mechanics. Different quantization procedures suggest that to have quantum probability from a classical system we have to introduce classical probability in a geometrical setup. Nelson's stochastic quantization method was expanded upon in a recent research [2] to include a relativistic framework and quantization of a Fermi field that accounts for Brownian motion processes in the interior space in addition to the exterior space. Anisotropy must be introduced into the interior space for a Fermi field to be quantized, allowing the internal variable to be seen as a direction vector connected to the outward space-time point. The geometrical and topological characteristics that emerge in an (3+ 1) - dimensional space-time when a direction vector or vortex line is connected to a space-time point will be reviewed in Sec. along with how they affect a (2+ 1)-dimensional system.

2. THEORETICAL BACKGROUND

Particle and antiparticle are represented by the direction vector's opposing orientations, respectively. To be equivalent to the Feynman path integral we have to take into account a complexified space-time when the coordinate is given by $Z_\mu = \chi_\mu + i\zeta_\mu$ where ζ_μ, corresponds to a direction vector attached to the space-time point χ_μ [3] . Since for quantization we have to introduce Brownian motion processes both in the external and internal space, after quantization, for an observational procedure we can think of the mean position of the particle q_μ in the external observable space with a stochastic extension determined by the internal stochastic variable ζ_μ . In the sharp point limit, the nonrelativistic quantum mechanics are achieved [4].. It has been shown that when we consider the internal space anisotropic in nature so that ζ_μ , appears as a direction vector, we can generate two internal helicities in terms of two spinorial variables giving rise to fcrmion and antifermion [3]. This helps us to have a gauge theoretic extension of a relativistic quantum particle when the gauge group is given by $SL(2,C)$. It appears that the primary component of the quantization process is this innate gauge structure. Recently Klauder [5] has formulated a quantization procedure that has the explicit property of coordinate independence. This is highly relevant as it clarifies how geometry functions in quantum probability. When quantization corresponds to freezing the particles at their initial Landau level, this geometrical approach to the phase space quantization may be understood in terms of a global magnetic field acting on a free particle in a higher dimension configuration space.In this approach, the critical role of the gauge field then emerges and leads to the equivalence of Klauder and stochastic quantization. The physical interpretation of the Hermitian line bundle, which was first presented in [6], may be found in the interaction between the magnetic field and the intrinsic gauge field in the formulation of stochastic phase space or in Klauder quantization, so the geometric quantization follows naturally from these two formalisms.

The location and momentum variables of a quantum particle can be expressed as follows, according to this interpretation of the stochastic quantization process

$$Q_\mu = q_\mu + i\hat{Q}_\mu \; ; \quad P_\mu = p_\mu + i\hat{P}_\mu \qquad (1)$$

where $q_\mu(p_\mu)$ denotes the mean position (momentum) in the external observable space and $Q_\mu(p_\mu)$ is given by the internal variable denoting the stochastic extension. Formulating the commutation relations is aided by the observation of Heisenberg's uncertainty relation from stochastic mechanics and the standard time energy uncertainty relation

$$[Q_\mu, P_\nu] = i\hbar g_{\mu\nu} , \quad [Q_\mu, Q_\nu] = 0 = [P_\mu, P_\nu] \qquad (2)$$

This suggests

$$[\hat{Q}_\mu, \hat{P}_\nu] = i\hbar g_{\mu\nu} , \quad [\hat{Q}_\mu, \hat{Q}_\nu] = 0 = [\hat{P}_\mu, \hat{P}_\nu] \qquad (3)$$

The components of the internal variables in this case will not commute, so the relations (2) and (3) will not be satisfied. On the other hand, as we have pointed out, the quantization of a Fermi field is achieved when we introduce an anisotropic feature in the internal space so that we can have two opposite internal helicities corresponding to fermion and antifermion. Indeed. We will have

$$[Q_\mu, Q_\nu] \neq 0, [P_\mu, P_\nu] \neq 0$$
$$[\hat{Q}_\mu, \hat{Q}_\nu] \neq 0, [\hat{P}_\mu, \hat{P}_\nu] \neq 0 \qquad (4)$$

Introducing a new constant $\omega = \hbar / lmc$, where m is the mass of the particle, the quantum uncertainty relation can now be written in terms of the dimensionless variables, where we replace Q_μ by Q_μ / l and P_μ by P_μ / mc

$$[Q_\mu, P_\nu] = i\omega g_{\mu\nu} \;\; ; [\hat{Q}_\mu, \hat{P}_\nu] = i\omega g_{\mu\nu} \qquad (5)$$

As has been shown by Brooke and Prugovecki [7], these relativistic canonical commutation relations admit the following representation of Q_μ / ω and P_μ / ω

$$Q_{\mu/\omega} = -i(\frac{\partial}{\partial p_\mu} + \phi_\mu),$$

$$P_{\mu/\omega} = i(\frac{\partial}{\partial q_\mu} + \psi_\mu) \qquad (6)$$

Where ϕ_μ and ψ_μ are complex-valued functions. Now, when we introduce an anisotropy in the internal space giving rise to the internal helicity to quantize a fermion. ϕ_μ and ψ_μ, became matrix-valued functions due to the noncommutativity character of the components $Q_\mu(P_\mu)$.

When we consider that the two opposite orientations of the direction vector ζ_μ attached to the space-time point χ_μ, in complexified Minkowski space-time having the coordinate ζ_μ^2 , give rise to two opposite intemal helicities corresponding to fermion and antifermion, we can formulate the internal helicity in terms of the two component spinorial variable $\theta(\bar{\theta})$ [8]. In fact for a massive spinor, we can choose the chiral coordinate in this space as

$$Z^\mu = \chi^\mu + (i/2)\lambda_\alpha^\mu \theta^\alpha \qquad (\alpha = 1,2) \qquad (7)$$

where the coordinate in the complex manifold $Z^\mu = \chi^\mu + i\zeta^\mu$ with $\zeta^\mu = \frac{1}{2}\lambda_\alpha^\mu \theta^\alpha$ is identified. We can replace the chiral coordinate by the matrices

$$Z^{AA'} = x^{AA'} + (i/2)\lambda_{\alpha}^{AA'}\theta^{\alpha}, x^{AA'} = \begin{bmatrix} x^0 - x^1 & x^2 + ix^3 \\ x^2 - ix^3 & x^0 + x^1 \end{bmatrix} \quad (8)$$

and

$$\lambda_{\alpha}^{AA'} \in SL(2,c)$$

The twistor equation is now changed in light of these relations as

$$\bar{Z}_a Z^a + \lambda_{\alpha}^{AA'}\theta^{\alpha}\bar{\pi}_A\pi_{A'} = 0 \quad (9)$$

where $\bar{\pi}_A\pi_{A'}$ is the spinorial variable corresponding to the four-momentum variable P^{μ} the conjugate of x^{μ} and is given by the matrix representation

$$P^{AA'} = \bar{\pi}^A\pi^{A'} \quad (10)$$

and

$$Z^a = (\omega^A, \pi_{A'}),\ \bar{Z}^a = (\bar{\pi}_A, \bar{\omega}^{A'})$$

with

$$\omega^A = i[x^{AA'} + (i/2)\lambda_{\alpha}^{AA'}\theta^{\alpha}]\pi_{A'}$$

Equation (9) now involves the helicity operator

$$s = -\lambda_{\alpha}^{AA'}\theta^{\alpha}\bar{\pi}_A\pi_{A'} \quad (11)$$

which we identify as the internal helicity of the particle and it corresponds to the fermion number. It may be noted that we have taken the matrix representation of P^{μ} the conjugate of x^{μ} in the complex coordinate $Z^{\mu} = \chi^{\mu} + i\zeta^{\mu}$ as $P^{AA'} = \bar{\pi}^A\pi^{A'}$ implying $P_{\mu}^2 = 0$ and so the particle will have mass due to the nonvanishing character of the quantity ζ_{μ}^2. It is observed that the complex conjugate of the chiral coordinate will give rise to a massive particle with opposite internal helicity corresponding to antifermion. In the null plane we can write the chiral coordinates as follows [9] .

$$Z^{AA'} = x^{AA'} + (i/2)\bar{\theta}^A\theta^{A'} \quad (12)$$

where the coordinate ζ^{μ} is replaced by $\zeta^{AA'} = \frac{1}{2}\bar{\theta}^A\theta^{A'}$. The helicity operator in this instance is provided by

$$s = -\bar{\theta}^A\theta^{A'}\bar{\pi}_A\pi_{A'} = -\bar{\in}\in \quad (13)$$

where $\in = i\theta^{A'}\pi_{A'}$, $\bar{\in} = -i\bar{\theta}^A\bar{\pi}_A$. The corresponding twistor equation describes a massless spinor field. The state with the helicity $\frac{1}{2}$ is the vacuum state of the fermion operator

$$\in \left| s = +\frac{1}{2} \right\rangle = 0 \quad (14)$$

Parallel to this, the fermion operator's vacuum state is the state with internal helicity $-\frac{1}{2}$.

$$\in \left| s = +\frac{1}{2} \right\rangle = 0 \qquad (15)$$

In the case of a massive spinor we can define a plane D^{-} where for the coordinate $Z_\mu = \chi_\mu + i\zeta_\mu$, where ζ_μ belongs to the interior of the forward light cone ζ □ 0 and represents the upper half plane. The lower half plane D^{+} is given by the set of all coordinates Z_μ with ζ_μ in the interior of the backward light cone ζ □ 0. The map $Z \rightarrow Z^{*}$ sends the upper half plane to the lower half plane. The space M of null plane $(\zeta_\mu^2 = 0)$ is the Shilov boundary so that a function holomorphic in D^{-} (D^{+}) is determined by its boundary values. Thus if we consider that any function $\phi(z) = \phi(x) + i\phi(\xi)$ is holomorphic in the whole domain, the helicity $+\frac{1}{2}(-\frac{1}{2})$ in the null plane may be taken to be the limiting value of internal helicity in the upper (lower) half plane. The domain with the properties ζ □ 0 and ζ □ 0 and in the upper and lower half planes suggests that the domain is disconnected in the sense of Minkowski space-time. This indicates that an angular momentum operator in such a region will behave similarly to a charged particle travelling through a magnetic monopole's field. In fact the wave function $\phi(z_\mu) = \phi(x_\mu) + i\phi(\xi_\mu)$ can be treated to describe a particle moving in the external space-time having the coordinate x_μ with an attached direction vector ξ_μ. Thus $\phi(z_\mu)$ should take into account the polar coordinates r, θ, ϕ along with the angle χ specifying the rotational orientation around the direction vector ξ_μ. The eigenvalue of the operator $i\frac{\partial}{\partial \chi}$ just corresponds to the internal helicity $+\frac{1}{2}(-\frac{1}{2})$. The three "Euler angles" are simply represented by $SO(4,1)$, θ, ϕ, χ for an extended body represented by the De-Sitter group. The angular momentum in this space is given by $J = r \times p - \mu r$ where μ is the eigenvalue of $i\frac{\partial}{\partial \chi}$ and can take the value $\mp\frac{1}{2}$. This implies that a particle can travel with $l = \frac{1}{2}$ in such a space. The fact that in such an anisotropic space the angular momentum can take the value $\frac{1}{2}$ is found to be analogous to the result that a monopole charged particle composite repre-senting a dyon satisfying the condition $e\mu = \frac{1}{2}$ have their angular momentum shifted by $\frac{1}{2}$ unit and their statistics shift accordingly [10].

In the complexified space-time exhibiting the internal helicity states we can now write the metric as $g_{\mu\nu}(x, \theta, \bar{\theta})$. It has been shown elsewhere [11] that the metric structure gives rise to the SL(2,c) gauge theory of gravitation and generates the field strength tensor $F_{\mu\nu}$, given in terms of gauge fields B_μ which are matrix valued having the SL(2,c) group structure and is given by

$$F_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu + [B_\mu, B_\nu] \qquad (16)$$

So from relations (6), we can identify ϕ_μ with B_μ and can associate another gauge field C_μ, with ψ_μ, satisfying the relation (16). This suggests that for a relativistic quantum particle which is taken as a stochastically extended one, a particle's fermionic nature links matrix-valued non-Abelian gauge fields with the SL(2,c) group structure to functions defined on stochastic phase space. That is, we write

$$\frac{Q_\mu}{\omega} = -i\left(\frac{\partial}{\partial p_\mu} + B_\mu\right),$$

$$\frac{P_\mu}{\omega} = i\left(\frac{\partial}{\partial q_\mu} + C_\mu\right) \qquad (17)$$

The asymptotic zero curvature condition $F_{\mu\nu} = 0$ implies that we can write the non-Abelian gauge field on the boundary as

$$B_\mu = U^{-1}\partial_\mu U, \qquad U \in SL(2,c) \qquad (18)$$

With this substitution, we note that the term $F_{\mu\nu}F^{\mu\nu}$ in the Lagrangian gives rise to the skyrme term $Tr[\partial_\mu UU^+, \partial_\mu UU^+]^2$ so that we can write the skynne Lagrangian

$$L = M^2 B_\mu B^\mu Tr(\partial_\mu U^+, \partial_\mu U) + Tr[\partial_\mu UU^+, \partial_\mu UU^+]^2 \quad (19)$$

where the first term can be derived from the term like $M^2 B_\mu B^\mu$ where M is a suitable constant having the dimension of mass. Thus we find that the quantization of a Fermi field considering an anisotropy in the internal space leading to an internal helicity description corresponds to the realization of a nonlinear α -model where the skyrme term ($L_{skyrme} = Tr[\partial_\mu UU^+, \partial_\mu UU^+]^2$) intro-duced for stabilization of the soliton automatically arises here as an effect of quantization. This research suggests that the fermion number has topological origin and that huge fermions arise as solitons. In fact, we may take the group manifold for the Hermitian representation as SU(2), which results in a mapping from the space three sphere s^3 to the group space $s^3[sU(2) = s^3]$. The associated winding number is provided by

$$q = \frac{1}{24\pi^2}\int ds_\mu \in^{\mu\nu\alpha\beta} Tr(U^{-1}\partial_\nu UU^{-1}\partial_\alpha UU^{-}\partial_\beta U) \qquad (20)$$

Evidently q can be taken to represent the fermion number.

It is to be noted that the simplest Lagrangian density which is invariant under SL(2,c) trans-formation in spinor affine space is given by

$$L = -\frac{1}{4} Tr \in^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} \qquad (21)$$

Following Carmeli and Malin[12] if we apply the usual procedure of variational calculus, we get the field equations

$$\partial_\delta(\in^{\alpha\beta\gamma\delta} F_{\alpha\beta}) - [B_\delta, \in^{\alpha\beta\gamma\delta} F_{\alpha\beta}] = 0 \qquad (22)$$

Taking the infinitesimal generators of the group SL(2,c) in the tangent space [12] as

$$g^1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, g^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \; g^3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad (23)$$

We can write

$$B_\mu = l_\mu^a g^a = b_\mu.g \qquad , F_{\mu\nu} = f_{\mu\nu}^a g^a = f_{\mu\nu}.g \qquad (24)$$

Thus to describe a matter field in this geometry the total Lagrangian will be modified by the introduction of this SL(2,c) invariant Lagrangian density. Thus, we write for the Lagrangian of a massless Dirac field

$$L = -\bar{\psi}\gamma_\mu D_\mu \psi - \frac{1}{4} Tr \in^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} \qquad (25)$$

where $D_\mu = \partial_\mu - igB_\mu$, being a suitable coupling strength. From this a conserved current is constructed [7]

$$J_\mu = -\bar{\psi}\gamma_\mu \psi + \in^{\mu\nu\alpha\beta} b_\nu \times f_{\alpha\beta} = J_x^\mu + J_0^\mu \qquad (26)$$

From (2) it follows that

$$\in^{\mu\nu\alpha\beta} (\partial_\nu \times F_{\alpha\beta} - b_\nu \times f_{\alpha\beta}) = 0 \qquad (27)$$

This suggests that

$$J_\theta^\mu = \in^{\mu\nu\alpha\beta} b_\nu \times f_{\alpha\beta} = \in^{\mu\nu\alpha\beta} \partial\nu \, f_{\alpha\beta} \qquad (28)$$

However. in (1) if we split the Dirac massless spinor in chiral forms and identify the internal helicity with left (right) chirality corresponding to $\theta(\bar{\theta})$ we have the following conservation laws [13]

$$\partial_\mu [\frac{1}{2}(-ig\bar{\psi}_R \gamma_\mu \psi_R) + J_\mu^1] = 0$$

$$\partial_\mu[\frac{1}{2}(-ig\bar{\psi}_L\gamma_\mu\psi_L + ig\bar{\psi}_R\gamma_\mu\psi_R) + J_\mu^2] = 0$$

$$\partial_\mu[\frac{1}{2}(-ig\bar{\psi}_L\gamma_\mu\psi_L) + J_\mu^2] = 0 \qquad (29)$$

These three equations represent a consistent set of equations if we choose

$$J_\mu^1 = -\frac{1}{2}J_\mu^2 \ , \ J_\mu^3 = +\frac{1}{2}J_\mu^2 \qquad (30)$$

which evidently gurantees the vector current conservation. Then we can write

$$\partial_\mu(\bar{\psi}_R\gamma_\mu\psi_R + J_\mu^2) = 0$$

$$\partial_\mu(\bar{\psi}_L\gamma_\mu\psi_L - J_\mu^2) = 0 \qquad (31)$$

$$\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = \partial_\mu J_\mu^5 = -2\partial_\mu J_\mu^2 \qquad (32)$$

Thus the chiral anomaly is expressed here in terms of the second $SL(2,C)$ component of the gauge field current J_μ^2. We note that chiral currents are modified by the introduction of J_μ^2 and the anomaly vanishes.

It is observed that the charge associated with the gauge field portion has the following form

$$q = \int J_\nu^2 d^3x = \int_{surface} \in^{ijk} d\sigma_i f_{jk}^2 \qquad (i,j,k = 1,2,3) \qquad (33)$$

Visualizing f_{jk}^2 to be magnetic field-like components for the vector potential b_i^2 we see that q is actually associated with the magnetic strength. It may be added here that the association of chiral anomaly with the Berry phase suggests that this topological phase $e^{i\phi_\beta}$ is given by the relation $\phi_\beta = 2\pi\mu$, where $q = 2\mu$ and is related to the fermion number [14].

Thus we find that the quantization of a Fermi field associates a background magnetic field corresponding to f_{ij}^2 and the charge corresponding to the gauge field effectively represents a magnetic charge.

The term $\in^{\alpha\beta\gamma\delta} TrF_{\alpha\beta}F_{\gamma\delta}$ in the Lagrangian (25) can be actually expressed as a four diver-gence $\partial_\mu\Omega_\mu$

$$\Omega^\mu = -\frac{1}{16\pi^2} \in^{\mu\nu\alpha\beta} Tr[B_\nu F_{\sigma\beta} - \frac{2}{3}(B_\nu B_\alpha B_\beta)] \qquad (34)$$

We acknowledge that the Pontryagin density and the gauge field Lagrangian are connected

$$P = -\frac{1}{16\pi^2} Tr * F_{\mu\nu}F^{\mu\nu} = \partial_\mu\Omega^\mu \qquad (35)$$

where Ω^{μ} is the Chem-Simons secondary characteristic class. The Pontryagin index $q = \int P d^4 x$ is a topological invariant.

The introduction of the Chem-Simons term modifies the axial vector current as $\tilde{J}_{\mu}^{5} = J_{\mu}^{5} + i\hbar\Omega_{\mu}$ where $\partial_{\mu}\tilde{J}_{\mu}^{5} = 0$ though $\partial_{\mu}\tilde{J}_{\mu}^{5} \neq 0$. We find from Eq. (32) that the Chem-Simons term is effectively represented by the current constructed from the SL(2,c) gauge field [15-22]. Thus we have Chem-Simons term effectively in built in the system and is associated with the topological aspects of the fermion arising out of the quantization procedure. From this analysis, we find that the non-Abelian gauge field associates a fictitious magnetic field in 3+1 dimension, and the introduction of the Pontryagin term $(\frac{\theta}{32\pi^2}) \in^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$ effectively takes care of the anisotropic feature of the space-time when a direction vector (vortex line) is attached to the space-time point.
In a 2+ 1 dimension the Hopf invariant is defined as

$$H = -\frac{1}{4\pi} \int d^3 x \in^{\hat{\mu}\hat{\nu}\hat{\lambda}} A_{\hat{\mu}} F_{\hat{\nu}\hat{\lambda}} \tag{36}$$

Now if μ denotes a four-dimensional index then

$$\partial\rho \in^{\rho\mu\nu\lambda} A_{\mu} F_{\nu\lambda} = \frac{1}{2} \in^{\rho\mu\nu\lambda} F_{\rho\mu} F_{\nu\lambda}$$

connects the Hopi invariant to the chiral anomaly. So from the above analysis we note that the Pontryagin term is associated with the Hopf term in a (2+1)-dimensional system. Non-Abelian gauge theories in 2+1 dimension with the incorporation of the Chem-Simons term

$$\frac{\mu}{2} Tr \in^{\mu\nu\lambda} [A_{\mu} F_{\nu\lambda} - \frac{2}{3}(A_{\mu} A_{\nu} A_{\lambda})] \tag{37}$$

represent the effect of a magnetic monopole field with the pole strength μ [23] The fact that a three-dimensional manifold B can be considered as a boundary of the four-dimensional manifold $M\,(\partial M = B)$ implies that the topological operation viz., the Pontryagin term, resulting from quantization in four dimensions, has a three-dimensional analog known as the Chem-Simons action. This relationship may be understood as follows

$$\int_{M_4} F \wedge F = \int_{M_3} (A \wedge dA + \frac{2}{3} A \wedge A \wedge A) \tag{38}$$

This implies that the same geometrical property which is responsible for the Pontryagin term in 3+ 1 dimension induces the Chem-Simons term in a 2+ 1 dimension [24].

Thus we find that both in 3+ 1 and 2+ 1 dimensions the origin of the Pontryagin term and the Chem-Simons term, respectively, associates an inherent magnetic monopole like behavior which manifests its properties through the anisotropic feature of space-time when a direction vector (vortex line) is attached to a space-time point [25-26].

3. CONCLUSION

The intrinsic Abelian gauge field structure in a nonAbelian gauge field theory in and in dimensions was covered in this study together with its geometrical and topological aspects [27]. It has been noticed that in and in dimensions the same geometrical characteristic is responsible for the formation of topological terms in the non-Abelian gauge field Lagrangian.

4. DISCUSSION

The Berry phase is realized because of this particular geometrical property. In the end, an anisotropic space's shape and the quantization of fermions within it produce a topological index that is comparable to the strength of a magnetic pole. According to this formalism, the fermion current, when divided into chiral form, aids in the formulation of electromagnetic interactions in disconnected form, indicating that the theory becomes asymptotically free in this scenario.

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